Apriori

- How to generate candidates?
  - Step 1: self-joining L_k
  - Step 2: pruning
- Example of Candidate-generation
  1. L_3 = {abc, abd, acd, ace, bcd}
  2. Self-joining $L_3 \otimes L_3$: abcd from abc and abd; acde from acd and ace
  3. Pruning: acde is removed because ade is not in L_3
  4. C_4 = {abcd}

Apriori

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

min_sup = 2

C_1

scan database for count of each candidate

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

L_1

compare candidate support count with min_sup

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

join and prune

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

C_2

scan database for count of each candidate

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

L_2

compare candidate support count with min_sup

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

join and prune

Itemset
{B, C, E}

C_3/L_3

scan database

Itemset	sup
{B, C, E}	2

Apriori

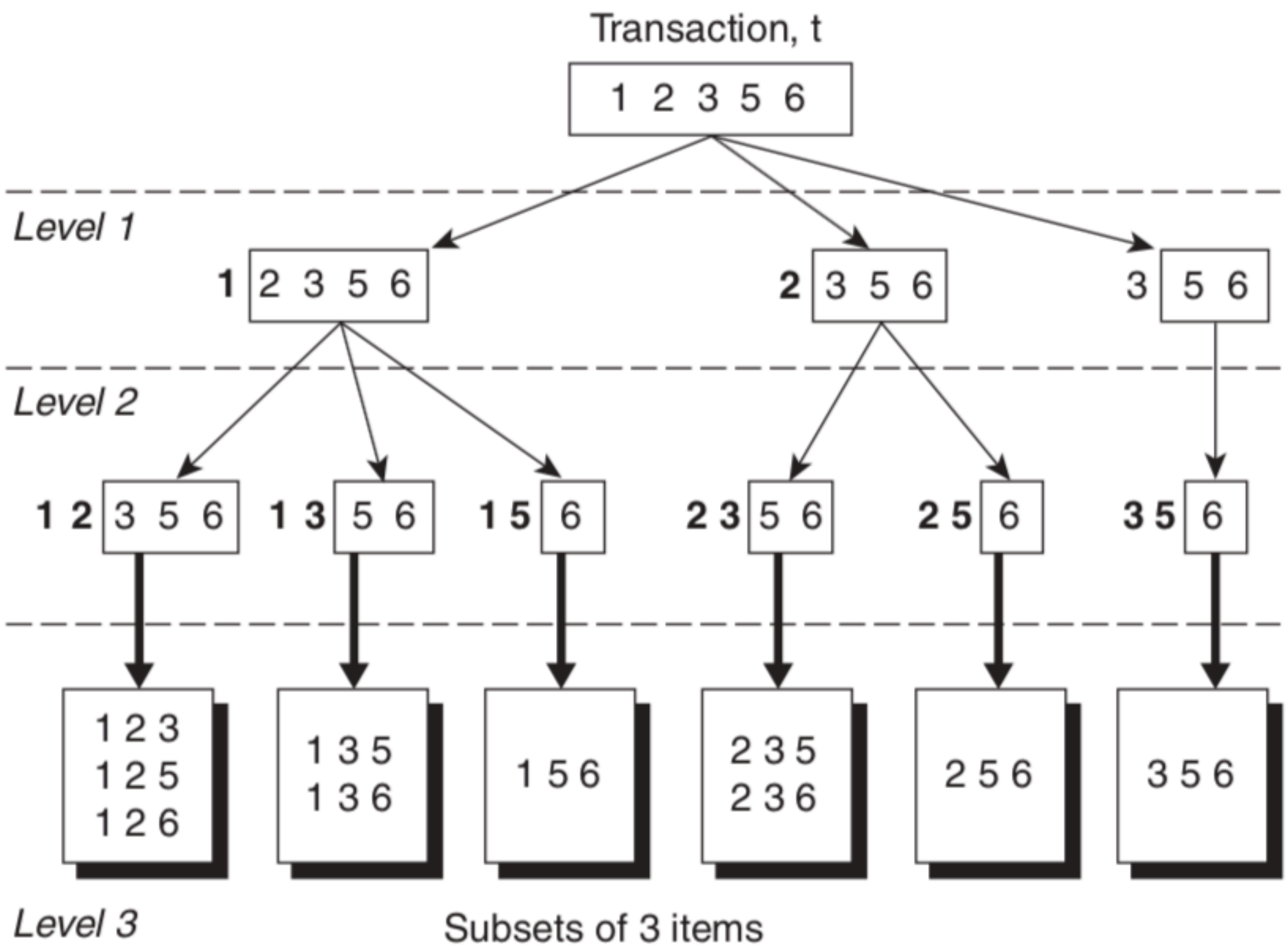
C_k: Candidate itemset of size k
L_k: Frequent itemset of size k

L_1 = {1-frequent items};
for (k = 1; L_k !=∅; k++) **do begin**
  C_{k+1} = candidates generated from L_k;
  for each transaction t in database do
    increment the count of all candidates in C_{k+1} that are contained in t
  end
  L_{k+1} = candidates in C_{k+1} with min_sup
end
return $\cup_k L_k$;

Apriori

- How to count supports of each candidate?
  - The total number of candidates can be huge
  - One transaction may contain many candidates
  - Support Counting Method:
    - store candidate itemsets in a hash-tree
    - leaf node of hash-tree contains a list of itemsets and counts
    - interior node contains a hash table

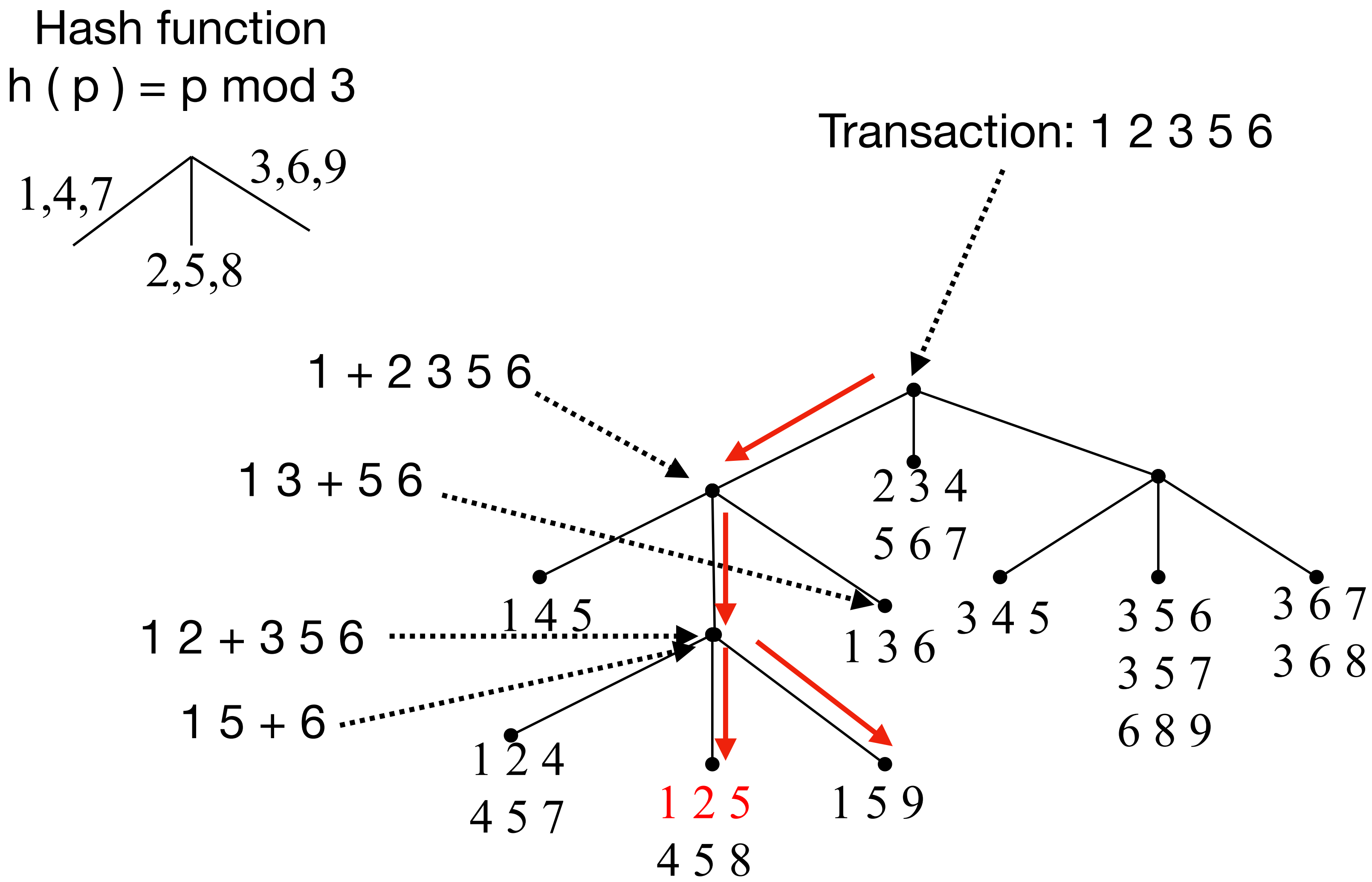
Apriori



Prefix structure enumerating 3-itemset in Transaction t

Figures from https://www-users.cs.umn.edu/~kumar001/dmbook/ch6.pdf

Apriori

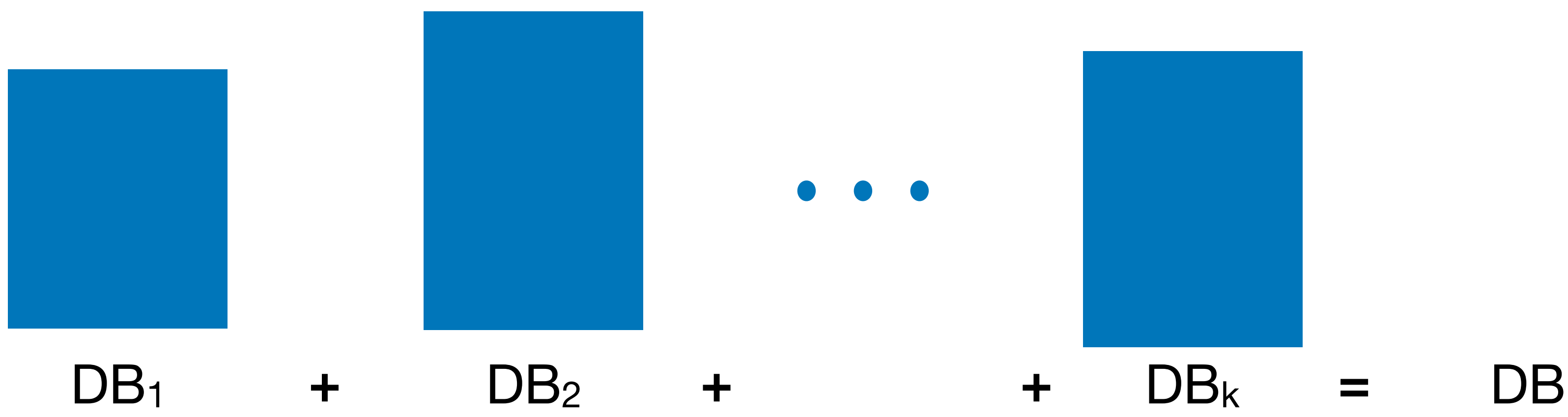

Hash function
h (p) = p mod 3
1,4,7
2,5,8
3,6,9
Transaction: 1 2 3 5 6
1 + 2 3 5 6
1 3 + 5 6
1 2 + 3 5 6
1 5 + 6
2 3 4
5 6 7
1 4 5
1 3 6
3 4 5
3 5 6
3 5 7
6 8 9
3 6 7
3 6 8
1 2 4
4 5 7
1 2 5
4 5 8
1 5 9

Improving the Efficiency of Apriori

- Challenges:
  - Multiple scans of transaction database
  - Huge number of candidates
  - Support counting for candidates
- Improving the Efficiency of Apriori
  - Reduce passes of transaction database scans
  - Shrink number of candidates
  - Facilitate support counting of candidates

Improving the Efficiency of Apriori

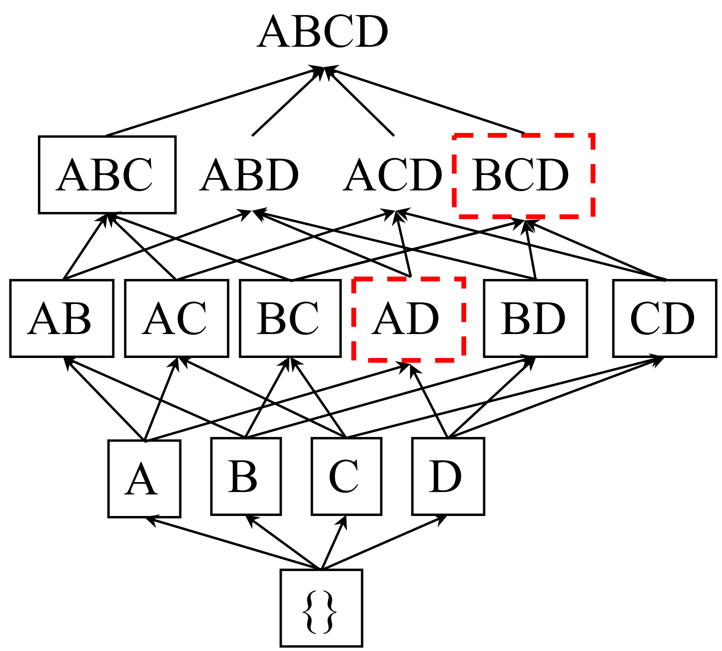
- Partition (reduce scans): partition data to find candidate itemsets
  - Any itemset that is potentially frequent (relative support ≥ min_sup) must be frequent (relative support in the partition ≥ min_sup) in at least one of the partition
    - Scan 1: partition database and find local frequent patterns
    - Scan 2: assess the actual support of each candidate to determine the global frequent itemsets

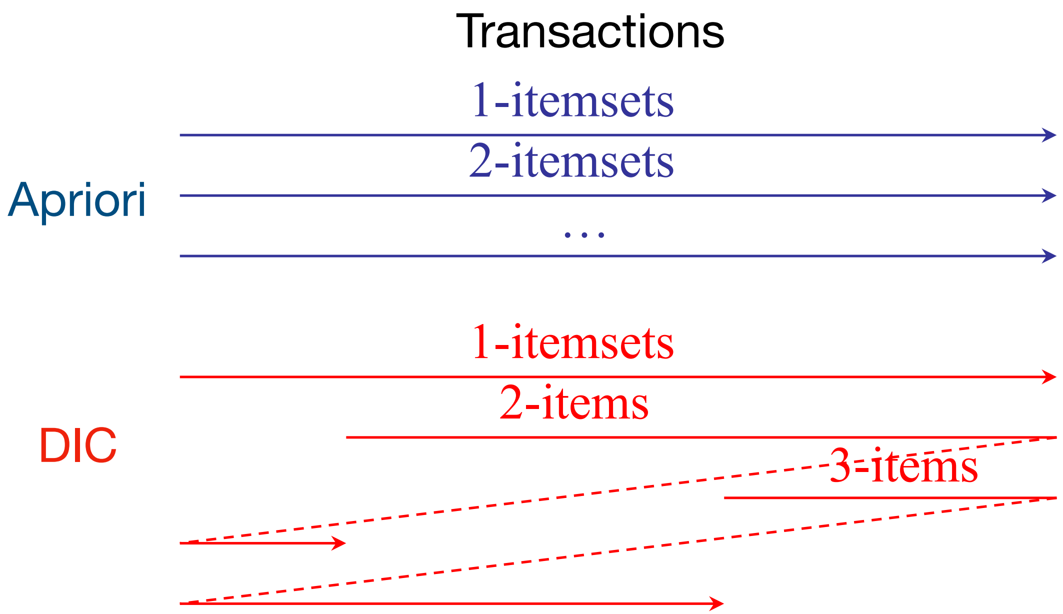


Improving the Efficiency of Apriori

- Dynamic itemset counting (reduce scans): adding candidate itemsets at different points during a scan

- new candidate itemsets can be added at any start point (rather than determined only before scan)





- once both A and D are determined frequent, the counting of AD begins

- Once all length 2 subsets of BCD are determined frequent, the counting of BCD begins

Improving the Efficiency of Apriori

- Hash-based Technique (shrink number of candidates): hashing itemsets into corresponding buckets
    - A k-itemset whose corresponding hashing bucket count is below min_sup cannot be frequent

min_sup = 3

Create hash table H_2 using hash function $h(x, y) = ((\mathit{order\ of\ } x) \times 10 + (\mathit{order\ of\ } y))\ \mathit{mod}\ 7$

H_2

bucket address	0	1	2	3	4	5	6
bucket count	2	2	4	2	2	4	4
bucket contents	{I1, I4}	{I1, I5}	{I2, I3}	{I2, I4}	{I2, I5}	{I1, I2}	{I1, I3}
	{I3, I5}	{I1, I5}	{I2, I3}	{I2, I4}	{I2, I5}	{I1, I2}	{I1, I3}
			{I2, I3}			{I1, I2}	{I1, I3}
			{I2, I3}			{I1, I2}	{I1, I3}

h(1, 4) = 1 * 10 + 4 = 0 mod 7
h(3, 5) = 3 * 10 + 5 = 0 mod 7

Improving the Efficiency of Apriori

- Sampling: mining on a subset of the given data
  - Trade off some degree of accuracy against efficiency
  - Select sample S of original database, mine frequent patterns within S (a lower support threshold) instead of the entire database —> the set of frequent itemsets local to S = L_S
  - Scan the rest of database once to compute the actual frequencies of each itemset in L_S
  - If L_S actually contains all the frequent itemsets, stop; otherwise
  - Scan database again for possible missing frequent itemsets

A Frequent-Pattern Growth Approach

- Bottlenecks of Apriori
  - Breadth-first (i.e., level-wise) search
  - Candidate generation and test, often generates a huge number of candidates
- FP-Growth
  - Depth-first search
  - Avoid explicit candidate generation
  - Grow long patterns from short ones using local frequent items
    - “abc” is a frequent pattern
    - Get all transactions having “abc,” i.e., project database D on abc: D | abc
    - “d” is a local frequent item in D | abc —> abcd is a frequent pattern

A Frequent-Pattern Growth Approach

TID	***Items bought***	***(ordered) frequent items***
100	***{f, a, c, d, g, i, m, p}***	***{f, c, a, m, p}***
200	***{a, b, c, f, l, m, o}***	***{f, c, a, b, m}***
300	***{b, f, h, j, o, w}***	***{f, b}***
400	***{b, c, k, s, p}***	***{c, b, p}***
500	***{a, f, c, e, l, p, m, n}***	***{f, c, a, m, p}***

min_sup = 3

F-list = f-c-a-b-m-p

1. Scan database once, find frequent 1-itemset
2. Sort frequent items in frequency descending order —> F-list

Header Table

Item	***frequency***	***head***
f	*4*	
c	*4*	
a	*3*	
b	*3*	
m	*3*	
p	*3*	

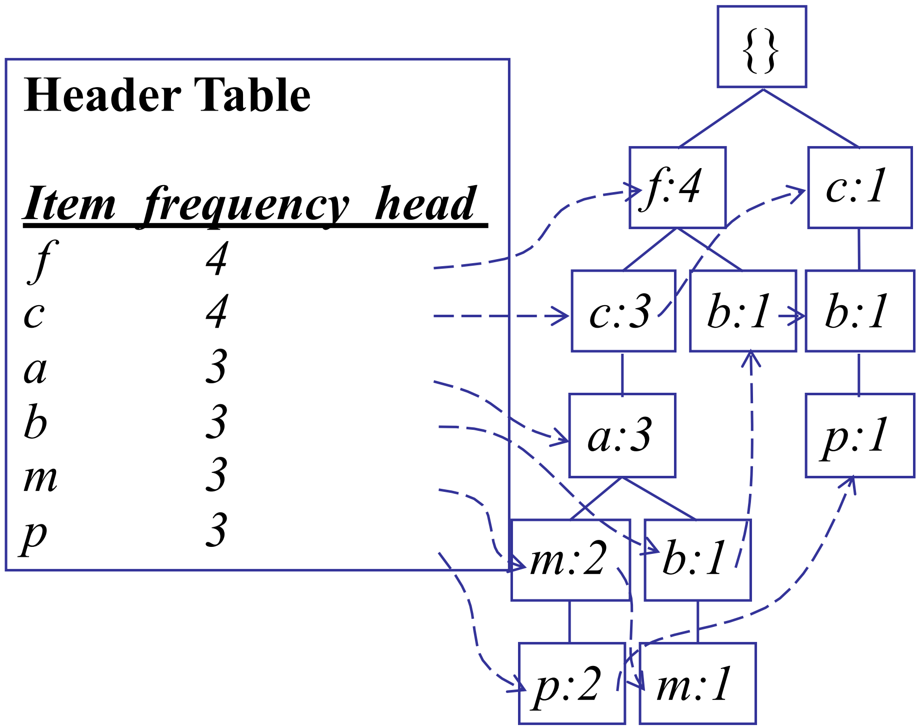
A Frequent-Pattern Growth Approach

TID	***Items bought***	***(ordered) frequent items***
100	***{f, a, c, d, g, i, m, p}***	***{f, c, a, m, p}***
200	***{a, b, c, f, l, m, o}***	***{f, c, a, b, m}***
300	***{b, f, h, j, o, w}***	***{f, b}***
400	***{b, c, k, s, p}***	***{c, b, p}***
500	***{a, f, c, e, l, p, m, n}***	***{f, c, a, m, p}***

min_sup = 3

F-list = f-c-a-b-m-p

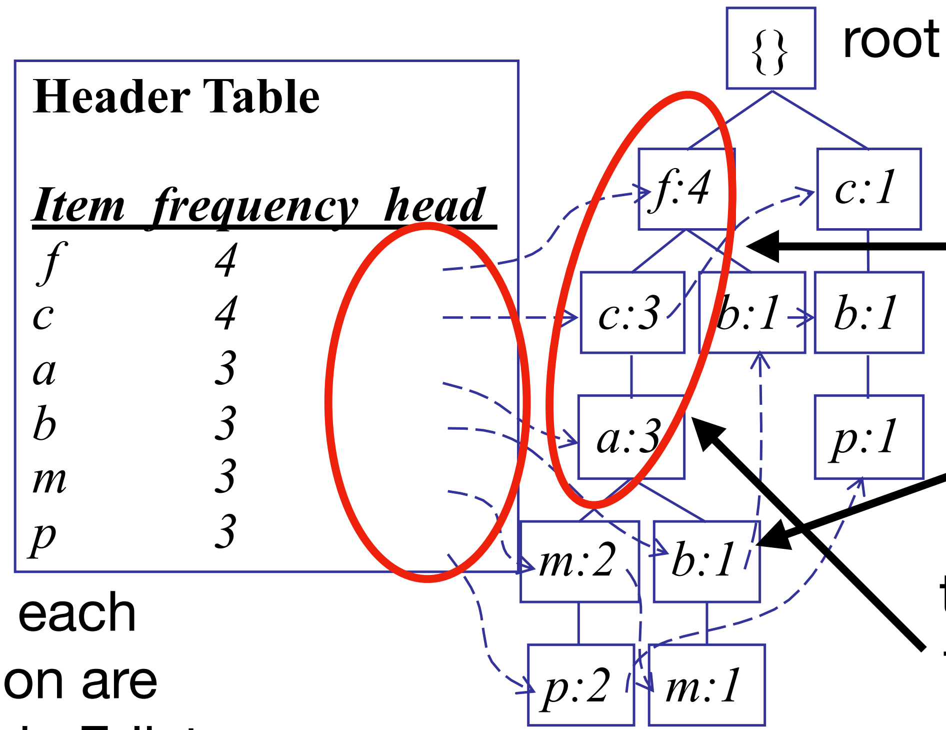
1. Scan database once, find frequent 1-itemset
2. Sort frequent items in frequency descending order —> F-list
3. Scan database again, construct FP-tree
4. Mine FP-tree



How to Construct FP-tree?

FP-tree: a compressed representation of database.
It retains the itemset association information.

To facilitate tree traversal, each item points to its occurrence in the tree via node-link



increment counts of existing nodes

create new nodes

two branches share the common prefix: f,c,a

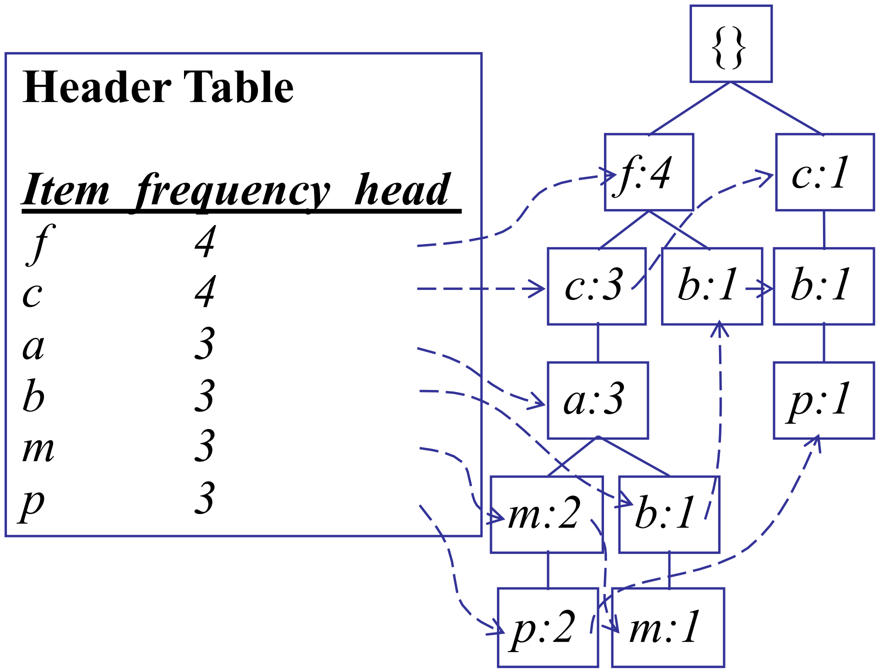
Items in each transaction are processed in F-list order

1st branch is created for transaction: f,c,a,m,p

2nd branch is created for transaction: f,c,a,b,m

How to Mine FP-tree?

1. Start from each frequent length-1 pattern (suffix pattern, usually the last item in F-list) to construct its conditional pattern base (prefix paths co-occurring with the suffix)

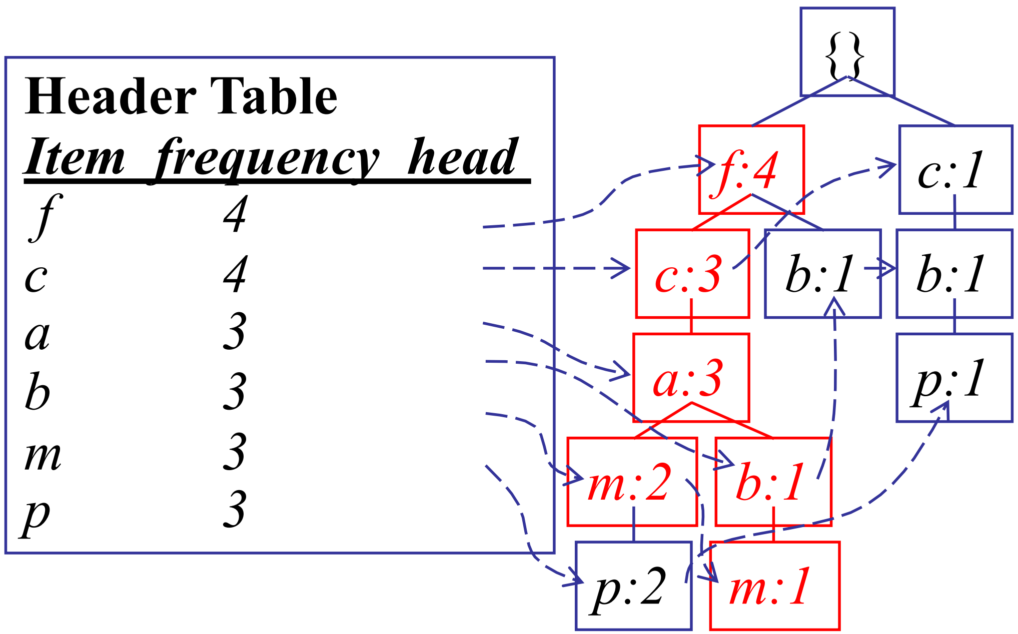


***Conditional* pattern bases**

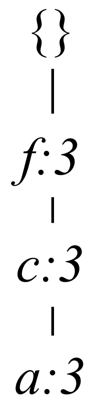
item	***cond. pattern base***
c	***f:3***
a	***fc:3***
b	***fca:1, f:1, c:1***
m	***fca:2, fcab:1***
p	***fcam:2, cb:1***

How to Mine FP-tree?

1. Start from each frequent length-1 pattern (suffix pattern, usually the last item in F-list) to construct its conditional pattern base
2. Construct the conditional FP-tree based on the conditional pattern base



m-conditional **pattern base:**
fca:2, fcab:1



m-conditional **FP-tree**

How to Mine FP-tree?

1. Start from each frequent length-1 pattern (suffix pattern, usually the last item in F-list) to construct its conditional pattern base
2. Construct the conditional FP-tree based on the conditional pattern base
3. Mining recursively on each conditional FP-tree until the resulting FP-tree is empty, or it contains only a single path — which will generate frequent patterns out of all combinations of its sub-paths

m-conditional **pattern base:**

fca:2, fcab:1

{}
|
f:3
|
c:3
|
a:3

m-conditional **FP-tree**

All frequent patterns relating to m: m, fm, cm, am, fcm, fam, cam, fcam

{}
|
f:3
|
c:3

am-conditional **FP-tree**

fc: 3

{}
|
f:3

cm-conditional **FP-tree**

f: 3

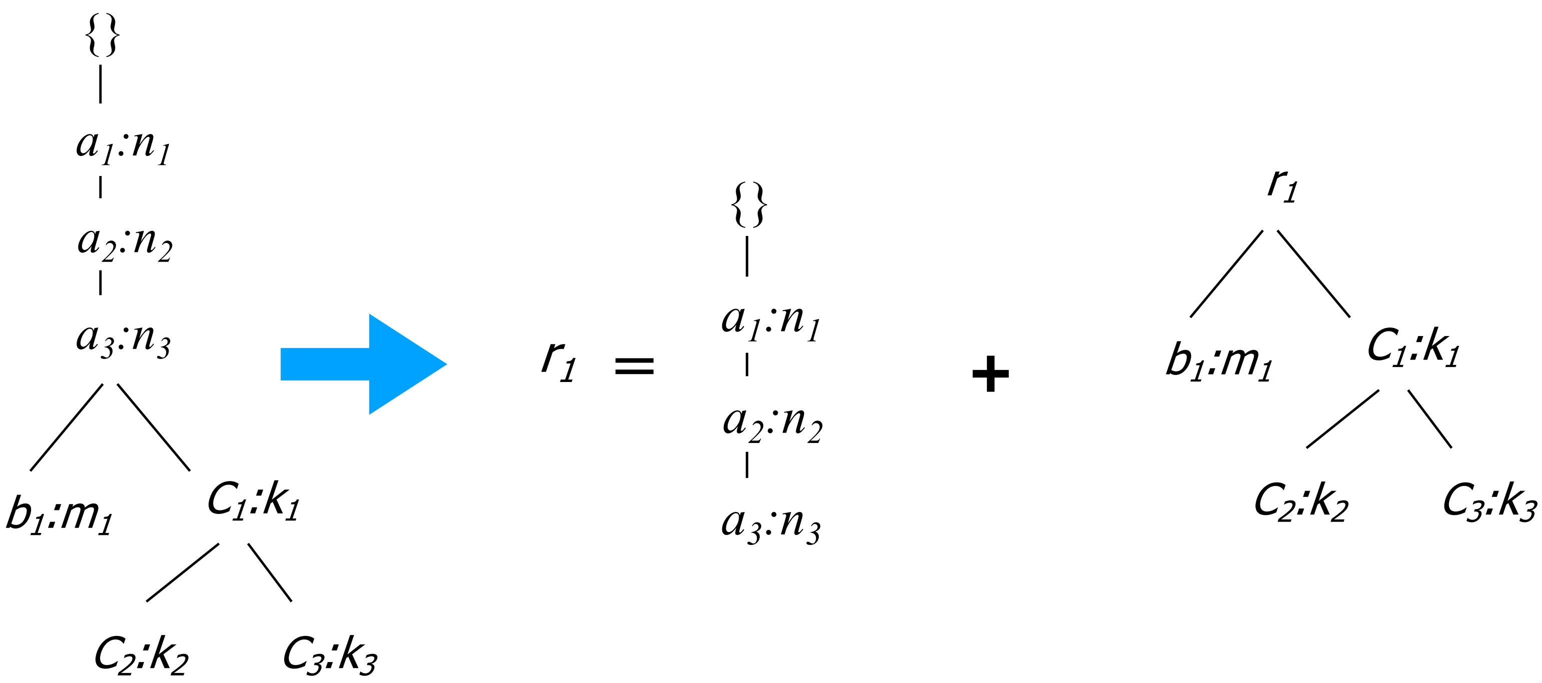
{}
|
f:3

cam-conditional **FP-tree**

f: 3

Single Prefix Path in FP-tree

- Suppose a (conditional) FP-tree has a shared single prefix-path
- Mining can be decomposed into two parts
  - Reduction of the single prefix path into one node
  - Concatenation of the mining results of the two parts



Scaling FP-Growth

- What if FP-tree cannot fit into memory?
  - Database projection: partition a database into a set of projected databases, then construct and mine FP-tree for each projected database
  - Parallel projection:
    - project the database in parallel for each frequent item
    - all partitions are processed in parallel
    - space costly
  - Partition projection:
    - project a transaction to a frequent item x if there is no any other item after x in the list of frequent items appearing in the transaction
    - a transaction is projected to only one projected database

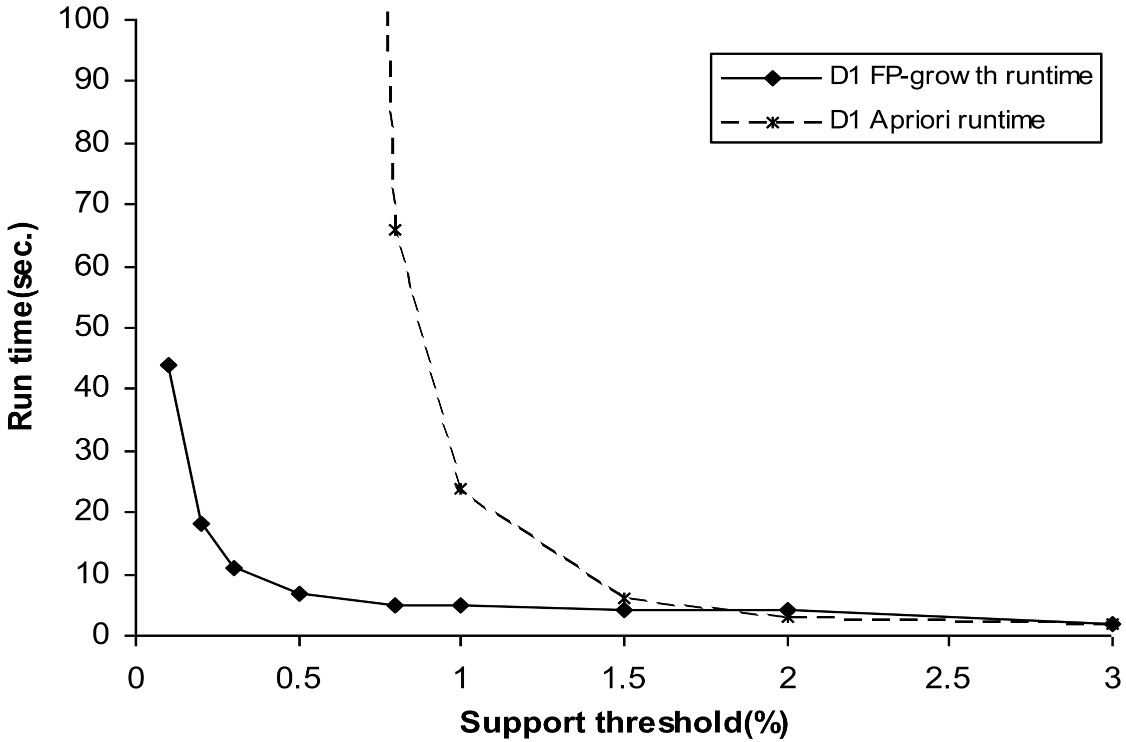
Benefits of FP-tree

- Completeness
  - Preserve complete information for frequent pattern mining
  - Never break a long pattern of any transaction
- Compactness
  - Reduce irrelevant info — infrequent items are gone
  - Items in frequency descending order: occurs more frequently, the more likely to be shared
  - Never be larger than the original database (not including node-links and the count fields)

Benefits of FP-Growth

- Divide-and-conquer:
  - Decompose both the mining task and database according to the frequent patterns obtained so far
  - Lead to focused search of smaller databases
- Other factors:
  - No candidate generation, no candidate test
  - Compressed database: FP-tree
  - No repeated scan of the entire database
  - Basic operations: count local frequent items and build sub FP-tree, no pattern search and matching

Performance of FP-Growth in Large Datasets



FP-Growth vs. Apriori

ECLAT: Frequent Pattern Mining with Vertical Data Format

- Vertical data format: itemset — transID_set
    - transID_set: a set of transaction IDs containing the itemset
- Derive frequent patterns based on the intersections of transID_set

itemset	*TID_set*
I1	{T100, T400, T500, T700, T800, T900}
I2	{T100, T200, T300, T400, T600, T800, T900}
I3	{T300, T500, T600, T700, T800, T900}
I4	{T200, T400}
I5	{T100, T800}

itemset	*TID_set*
{I1, I2}	{T100, T400, T800, T900}
{I1, I3}	{T500, T700, T800, T900}
{I1, I4}	{T400}
{I1, I5}	{T100, T800}
{I2, I3}	{T300, T600, T800, T900}
{I2, I4}	{T200, T400}
{I2, I5}	{T100, T800}
{I3, I5}	{T800}

itemset	*TID_set*
{I1, I2, I3}	{T800, T900}
{I1, I2, I5}	{T100, T800}

ECLAT: Frequent Pattern Mining with Vertical Data Format

- Vertical data format: itemset — transID_set
  - transID_set: a set of transaction IDs containing the itemset
- Derive frequent patterns based on the intersections of transID_set
- Use diffset to reduce the cost of storing long transID_set
  - {I1} = {T100, T400, T500, T700, T800, T900}
  - {I1, I2} = {T100, T400, T800, T900}
  - diffset({I1}, {I1, I2}) = {T500, T700}

Summary

- Frequent itemset mining methods:
  - Apriori: candidate generation-and-test
  - Improving efficiency of Apriori: partition, dynamic item counting, hash-based technique, sampling
  - FP-Growth: depth-first search
  - Scaling of FP-Growth: database projection
  - Frequent pattern mining with vertical data format

Outline

- Basic Concepts in Frequent Pattern Mining
- Frequent Itemset Mining Methods
- Pattern Evaluation Methods

Pattern Evaluation Methods: Correlations

- play basketball ⇒ eat cereal [40%, 66.7%] is misleading
    - the overall % of students eating cereal is 75% > 66.7%
- play basketball ⇒ not eat cereal [20%, 33.3%] is more accurate
- Lift: a measure of dependent/correlated event

$$\text{lift} = \frac{P(A \cup B)}{P(A)P(B)} = \frac{P(B|A)}{P(B)}$$

	Basketball	Not	Sum
Cereal	2000	1750	3750
Not cereal	1000	250	1250
Sum(col.)	3000	2000	5000

$$lift(Basketball, Cereal) = \frac{2000/5000}{(3000/5000) \times (3750/5000)} = 0.89$$

< 1, negatively correlated

$$lift(Basketball, Notcereal) = \frac{1000/5000}{(3000/5000) \times (1250/5000)} = 1.33$$

Other Pattern Evaluation Methods

- χ^2 measure, all_confidence measure, max_confidence measure, Kulczynski measure, …