Principles of Wireless Communication and Mobile Networks

IE 304

Game Theory in Mobile and Wireless Systems: Mechanism Design Basics

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Goal of Mechanism Design

Define the rules of a game so that in equilibrium rational and selfish agents do what we want.
A single item is auctioned to the bidders according to the rules:

- Each bidder privately submits a bid to the auctioneer (e.g., in a sealed envelope).
- The bidder with the highest bid (i.e., winner) gets the item.
- The winner pays the 2\textsuperscript{nd} highest bid.

**How much would you bid?**

Payoff $= 100¥ - 90¥$
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**Claim 1**: In a second-price auction, every bidder has a dominant strategy: set his bid to his private valuation.

**Claim 2**: In a second-price auction, every truth-telling bidder guaranteed non-negative utility.
- Examples of mechanisms
  - Auction
  - Voting: choose a candidate among a group
  - Public project: decide whether to build a swimming pool whose cost must be funded by the agents themselves
  - ...

- Mechanism design in mobile and wireless systems
  - Wireless spectrum auctions
  - Incentive mechanisms for crowd sensing systems
  - ...

Introduction
What properties should an awesome auction satisfy?

- Awesome auctions (William Vickrey)

How to design an auction with all the desirable properties?

- Myerson’s Lemma (Roger Myerson)
- VCG Mechanism (William Vickrey, Edward H. Clarke, Theodore Groves)
Theorem: An awesome auction enjoys three desirable properties:

- **Strong Incentive Guarantees**: It is dominant-strategy incentive-compatible (DSIC), i.e.,
  - Every bidder has a dominant strategy: set his bid to his private valuation.
  - Every truth-telling bidder guaranteed non-negative utility.

- **Strong Performance Guarantees**: If bidders report truthfully, then the auction maximizes the social surplus

\[ \sum_{i=1}^{n} v_i x_i, \]

where \( x_i = 1 \), if bidder \( i \) wins, and 0, if bidder \( i \) loses, subject to the feasibility constraint that \( \sum_{i=1}^{n} x_i \leq 1 \) (i.e., there is only one item).

- **Computational Efficiency**: The auction can be implemented in polynomial time.

**Why are these properties important?**
DSIC:
- Easy for a bidder to choose a bid.
- Easy to reason about the auction outcome.

Social Surplus Maximization:
- The auction identifies the bidder with the highest valuation, even though the bidder valuations were a priori unknown to the auctioneer!

Polynomial-Time Computational Efficiency:
- Practical implementations!
Myerson’s Lemma

- **Single-Parameter Environments:**
  - $n$ bidders.
  - Each bidder $i$ has a **private** valuation $v_i$.
  - A feasible set $\mathcal{X}$, where each element is a vector $(x_1, x_2, \ldots, x_n)$ and $x_i$ denotes the “amount of stuff” given to bidder $i$.
    - **Example 1:** In a single-item auction, $\mathcal{X}$ is the set of 0-1 vectors s.t. $\sum_{i=1}^{n} x_i \leq 1$.
    - **Example 2:** With $k$ identical goods and the constraint that each customer gets at most one, $\mathcal{X}$ is the set of 0-1 vectors s.t. $\sum_{i=1}^{n} x_i \leq k$. 
 Allocation and payment rules of a sealed-bid auction:

- Collect bids $\mathbf{b} = (b_1, b_2, \cdots, b_n)$.
- **Allocation Rule:** Choose a feasible allocation $\mathbf{x}(\mathbf{b}) \in \mathcal{X} \subseteq \mathbb{R}^n$ as a function of the bids.
- **Payment Rule:** Choose payments $\mathbf{p}(\mathbf{b}) \in \mathbb{R}^n$ as a function of the bids.

Bidder $i$’s utility on the bid profile $\mathbf{b}$:

$$u_i(\mathbf{b}) = v_i x_i - p_i(\mathbf{b}).$$
Implementable Allocation Rule:
- An allocation rule $x(\cdot)$ for a single-parameter environment is **implementable** if there is a payment rule $p(\cdot)$ such that the sealed-bid auction $(x, p)$ is DSIC.

Monotone Allocation Rule:
- An allocation rule $x(\cdot)$ for a single-parameter environment is **monotone** if for every bidder $i$ and bids $b_{-i}$ by the other bidders, the allocation $x_i(z, b_{-i})$ to $i$ is nondecreasing in its bid $z$. 
Myerson’s Lemma:

- **Myerson’s Lemma**: For a single-parameter environment
  
  a) An allocation rule \( x(\cdot) \) is implementable if and only if it is monotone.
  
  b) If \( x(\cdot) \) is monotone, then there is a unique payment rule \( p(\cdot) \) such that the sealed-bid auction \((x, p)\) is DSIC.
  
  c) The payment rule in b) is given by

\[
p_i(b_i, b_{-i}) = \sum_{j=1}^{l} z_j \cdot \text{jump in } x_i(\cdot, b_{-i}) \text{ at } z_j,
\]

where \( z_1, \ldots, z_l \) are the breakpoints of the allocation function \( x_i(\cdot, b_{-i}) \) in the range \([0, b_i]\).

Examples of

\[x_i(z) = x_i(z, b_{-i})\]
Application of Myerson’s Lemma in a single-item auction:

- **Allocation rule**: Allocate the item to the highest bidder.

\[ x_i(z) = x_i(z, b_{-i}) \]

- **Payment rule**?

\[ p_i(b_i, b_{-i}) = \begin{cases} 0, & \text{if } b_i \leq \max_{j \neq i} b_j \\ \max_{j \neq i} b_j, & \text{if } b_i > \max_{j \neq i} b_j \end{cases} \]

Second-price auction!
Example: The buying of multiple items

- Each \( i \) has a value \( v_i \) for an item.
- There are 5 items in total!
- Each bidder wants only one item.

What is an efficient outcome?
- Suppose bidders have valuations 
  $70 \$30 \$27 \$25 \$12 \$5 \$2$
  - Sell the items to the 5 bidders with the highest values

How to design the auction?
- A general design rule is the Vickery-Clarke-Grove (VCG) mechanism.
VCG Mechanism

- **Goal**: Implement the efficient outcome in dominant strategies.
- **VCG** is a general method generalizing second-price auctions.
- **Solution**: Each bidder pays the **damage** they impose on society.
- We can maximize efficiency by
  - Choosing the **efficient outcome** (given the bids) as allocation.
  - Each bidder pays his **social cost** (welfare).

\[
p_i = \text{Optimal welfare (for the other bidders), if bidder } i \text{ was not participating.} - \text{Welfare of the other bidders from the chosen outcome.}
\]
VCG Mechanism

- Assume that bids = valuations
  - $70 $30 $27 $25 $12 $5 $2

- Optimal welfare, if bidder \( i \) was not participating.
  - $99 $139 $142 $144 $157 $164 $164

- Welfare of the other players from the chosen outcome.
  - $94 $134 $137 $139 $157 $164 $164

- This gives payments
  - $5 $5 $5 $5 $5 $0 $0

- VCG rules for \( k \)-item auctions:
  - Highest \( k \) bids win an object.
  - The winners pay the \( (k + 1) \)th highest bid.

- Truthfulness is a dominant strategy.
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Limitations of VCG?

- What if maximizing social welfare is computationally intractable?