1. ([R], Page 689, Exercise 2(a)(b)(d)) Does each of these lists of vertices form a path in the following graph? Which paths are simple? Which are circuits? What are the lengths of those that are paths?
   a) a, b, e, c, b
   b) a, d, a, d, a
   d) a, b, e, c, b, d, a

   Answer Area:

2. ([R], Page 689, 6) How many connected components does each of the graphs in Exercises 3-5 have? For each graph find each of its connected components.

   Answer Area:
3. ([R], Page 690, 14(a)(b)(c)) Find the strongly connected components of each of these graphs.

a) 

\[ \begin{array}{c}
\text{a} & \text{b} & \text{c} \\
\text{e} & \text{d} & \\
\end{array} \]

b) 

\[ \begin{array}{c}
\text{a} & \text{b} & \text{c} \\
\text{f} & \text{e} & \text{d} \\
\end{array} \]

c) 

\[ \begin{array}{c}
\text{a} & \text{b} & \text{c} & \text{d} & \text{e} \\
\text{i} & \text{h} & \text{g} & \text{f} \\
\end{array} \]

Answer Area:

4. ([R], Page 692, Exercise 50(a)(c)(d)) For each of these graphs, find \( \kappa(G) \), \( \lambda(G) \), and \( \min_{v \in V} \deg(v) \), and determine which of the two inequalities in \( \kappa(G) \leq \lambda(G) \leq \min_{v \in V} \deg(v) \) are strict.

a) 

\[ \begin{array}{c}
\text{a} & \text{b} & \text{c} \\
\text{e} & \text{d} & \\
\end{array} \]

b) 

\[ \begin{array}{c}
\text{a} & \text{b} & \text{c} \\
\text{d} & \text{e} & \text{f} \\
\text{g} & \text{h} & \text{i} \\
\text{j} & \text{k} & \text{l} \\
\end{array} \]

c) 

\[ \begin{array}{c}
\text{a} & \text{b} & \text{c} \\
\text{d} & \text{e} & \text{f} \\
\end{array} \]

d) 

\[ \begin{array}{c}
\text{a} & \text{b} & \text{c} \\
\text{d} & \text{e} & \text{f} \\
\end{array} \]

Answer Area:
5. ([R], Page 692, Exercise 52(b)) Show that if $G$ is a connected graph with $n$ vertices then
   b) $\lambda(G) = n - 1$ if and only if $G = K_n$

   **Note:** Please prove without using the inequalities $\kappa(G) \leq \lambda(G) \leq \min_{v \in V} \{v\}$. You can proceed by induction on the number of vertices.

   **Answer Area:**

6. ([R], Page 692, Exercise 63) Show that a simple graph $G$ is bipartite if and only if it has no circuits with an odd number of edges.
   **Hint:** Fix any vertex $v$ and consider partition the vertices into those which can be connected from $v$ through a path of odd length and those which can be connected from $v$ through a path of even length.

   **Answer Area:**

7. ([R], Page 704, Exercise 6, 8) In Exercises 6, 8 determine whether the given graph has an Euler circuit. Construct such a circuit when one exists. If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path if one exists.
8. ([R], Page 705, Exercise 36) In Exercises 36 determine whether the given graph has a Hamilton circuit. If it does, find such a circuit. If it does not, give an argument to show why no such circuit exists.

Answer Area:
9. ([R], Page 705, Exercise 40) Does the graph in Exercise 33 have a Hamilton path? If so, find such a path. If it does not, give an argument to show why no such path exists.

33. [Diagram of a graph with vertices labeled a, b, c, d, e, f, g and edges connecting them.]

Answer Area:

10. ([R], Page 706, Exercise 49) Show that there is a Gray code of order \( n \) whenever \( n \) is a positive integer, or equivalently, show that the \( n \)-cube \( Q_n, n > 1 \), always has a Hamilton circuit. [Hint: Use mathematical induction. Show how to produce a Gray code of order \( n \) from one of order \( n - 1 \).]

Answer Area:

11. ([R], Page 706, Exercise 55) Show that a bipartite graph with an odd number of vertices does not have a Hamilton circuit.

Answer Area: