1. ([R], Page 725, Exercise 2, 4) In Exercises 2-4 draw the given planar graph without any crossings.

Answer Area:

2) 

4) 

2. ([R], Page 726, Exercise 17) Suppose that a connected planar simple graph with \( e \) edges and \( v \) vertices contains no simple circuits of length 4 or less. Show that \( e \leq \frac{5}{3}v - \frac{10}{3} \) if \( v \geq 4 \)

Answer Area:

As in the argument in the proof of Corollary 1, we have \( 2e \geq 5r \) and \( r = e - v + 2 \). Thus \( e - v + 2 \leq 2e/5 \), which implies that \( e \leq \frac{5}{3}v - \frac{10}{3} \).

3. ([R], Page 725, Exercise 7, 8, 9, Page 726, Exercise 23, 25) Judge whether the following simple graphs are planar or not:

If the graph is planar, present a planar drawing of the graph. Otherwise, use Kuratowski’s
Theorem to prove that it is not planar.

7) It’s a planar graph.

8) It’s not a planar graph. There exists a sub-graph that is $K_{3,3}$.

9) It’s not a planar graph. There exists a sub-graph that is homeomorphic to $K_{3,3}$.
23) It’s a planar graph.

25) It’s not a planar graph. There exists a sub-graph that is homeomorphic to $K_5$.

4. ([R], Page 733, Exercise 6) In Exercises 6 find the chromatic number of the given graph.

Answer Area:

Since there is a triangle, at least 3 colors are needed. To show that 3 colors suffice, notice that we can color the vertices around the outside alternately using red and blue, and color vertex $g$ green.