1. ([R], Page 557, Exercise 8) In a survey of 270 college students, it is found that 64 like brussels sprouts, 94 like broccoli, 58 like cauliflower, 26 like both brussels sprouts and broccoli, 28 like both brussels sprouts and cauliflower, 22 like both broccoli and cauliflower, and 14 like all three vegetables. How many of the 270 students do not like any of these vegetables?

**Answer Area:**

\[
270 - (94 + 58 + 64 - 28 - 22 - 26 + 14) = 116
\]

There’re 116 students don’t like any of these vegetables.

2. ([R], Page 558, Exercise 22) Prove the principle of inclusion-exclusion using mathematical induction.

**Answer Area:**

Let \( A_1, \ldots, A_n \) be finite sets. For \( n = 2 \), we know the principle to be valid. Suppose that the principle is true for \( n \) sets. Consider \( n + 1 \) sets and regard \( A_n \cup A_{n+1} \) as one set, then by the inductive hypothesis we have

\[
|A_1 \cup \ldots \cup A_{n+1}| = \sum_{i<n} |A_i| + |A_n \cup A_{n+1}| - \sum_{i<j<n} |A_i \cap A_j| - \sum_{i<n} |A_i \cap (A_n \cup A_{n+1})|
\]

By applying the distributive law to each term on the right involving \( A_n \cup A_{n+1} \), we get

\[
\sum |(A_{i_1} \cap \cdots \cap A_{i_m}) \cap (A_n \cup A_{n+1})| = \sum |(A_{i_1} \cap \cdots \cap A_{i_m} \cap A_n) \cup (A_{i_1} \cap \cdots \cap A_{i_m} \cap A_{n+1})|
\]

Thus, we can rewrite each of these terms as

\[
\sum |A_{i_1} \cap \cdots \cap A_{i_m} \cap A_n| + \sum |A_{i_1} \cap \cdots \cap A_{i_m} \cap A_{n+1}| - \sum |A_{i_1} \cap \cdots \cap A_{i_m} \cap A_n \cap A_{n+1}|
\]

3. ([R], Page 565, Exercise 13) How many derangements are there of a set with seven elements?

**Answer Area:**

\[
D_7 = 7! \left(1 - \sum_{i=1}^{7} (-1)^{i+1} \cdot \frac{1}{i!}\right) = 1854
\]

There’re 1854 derangements of a set with seven elements.
4. ([R], Page 565, Exercise 16) A group of $n$ students is assigned seats for each of two classes in the same classroom. How many ways can these seats be assigned if no student is assigned the same seat for both classes?

**Answer Area:**

There are $n!$ ways to make the first assignment about $n$ students. Then the next seating must be a derangement with respect to this numbering, so there are $D_n$ second seatings possible. And there are $n!D_n$ in all.

5. ([R], Page 565, Exercise 17) How many ways can the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 be arranged so that no even digit is in its original position?

**Answer Area:**

$$10! - (5 \cdot 9!) - \binom{5}{2} \cdot 8! + \binom{5}{3} \cdot 7! - \binom{5}{4} \cdot 6! + 5!) = 2170680$$

There’re 2170680 ways that no even digit is in its original position.