Coq 中的集合证明方法汇总

常用证明方法：

- 规约为逻辑命题证明，`Sets_unfold` （保留 `∈` 符号）与 `sets_unfold` （不保留 `∈` 符号）。
- 将集合相等规约为证明相互包含：`apply Sets_equiv_Sets_included; split` 。

```
Sets_equiv_Sets_included:
  forall x y, x == y <-> x ⊆ y /\ y ⊆ x
```

- 常用证明思路：分析处理 `⊆` 左右两侧的集合交与集合并。

  `x ⊆ y ∩ z` 可以被规约为 `x ⊆ y` 与 `x ⊆ z` ；
  `x ∩ y ⊆ z` 可以被规约为 `x ⊆ z` ；
  `x ∩ y ⊆ z` 也可以被规约为 `y ⊆ z` 。

```
Sets_included_intersect:
  forall x y z, x ⊆ y -> x ⊆ z -> x ⊆ y ∩ z
Sets_intersect_included1:
  forall x y, x ∩ y ⊆ x
Sets_intersect_included2:
  forall x y, x ∩ y ⊆ y
```

  `x ⊆ y ∪ z` 可以被规约为 `x ⊆ y` ；
  `x ⊆ y ∪ z` 也可以被规约为 `x ⊆ z` ；
  `x ∪ y ⊆ z` 可以被规约为 `x ⊆ z` 与 `y ⊆ z` ；
  无穷并的处理思路也类似。

```
Sets_included_union1:
  forall x y, x ⊆ x ∪ y
Sets_included_union2:
  forall x y, y ⊆ x ∪ y
Sets_union_included:
  forall x y z, x ⊆ z -> y ⊆ z -> x ∪ y ⊆ z;
Sets_included_indexed_union:
  forall n xs x, xs n ⊆ ⋃ xs
Sets_indexed_union_included:
  forall xs y, (forall n, xs n ⊆ y) -> ⋃ xs ⊆ y
```

- 常用定理列表 1（交集与并集）：

```
Sets_intersect_comm:
  forall x y, x ∩ y == y ∩ x
Sets_intersect_assoc:
  forall x y z, (x ∩ y) ∩ z == x ∩ (y ∩ z)
Sets_union_comm:
  forall x y, x ∪ y == y ∪ x
Sets_union_assoc:
  forall x y z, (x ∪ y) ∪ z == x ∪ (y ∪ z)
Sets_union_intersect_distr_l:
  forall x y z, x ∪ (y ∩ z) == (x ∪ y) ∩ (x ∪ z)
Sets_union_intersect_distr_r:
  forall x y z, (x ∩ y) ∪ z == (x ∪ z) ∩ (y ∪ z)
Sets_intersect_union_distr_r:
  forall x y z, (x ∪ y) ∩ z == x ∩ z ∪ y ∩ z
Sets_intersect_union_distr_l:
  forall x y z, x ∩ (y ∪ z) == x ∩ y ∪ x ∩ z
```

- 常用定理列表 2（无穷并集）：

```
Sets_intersect_indexed_union_distr_r:
 forall xs y, ⋃ xs ∩ y == ⋃ (fun n => xs n ∩ y)
Sets_intersect_indexed_union_distr_l:
 forall x ys, x ∩ ⋃ ys == ⋃ (fun n => x ∩ ys n)
```

- 常用定理列表 3（二元关系的连接）：

```
Rels_concat_assoc:
  forall x y z, (x ∘ y) ∘ z == x ∘ y ∘ z
Rels_concat_id_l:
  forall x, Rels.id ∘ x == x
Rels_concat_id_r:
  forall x, x ∘ Rels.id == x
Rels_concat_union_distr_l:
  forall x y1 y2, x ∘ (y1 ∪ y2) == x ∘ y1 ∪ x ∘ y2
Rels_concat_union_distr_r:
  forall x1 x2 y, (x1 ∪ x2) ∘ y == x1 ∘ y ∪ x2 ∘ y
Rels_concat_indexed_union_distr_l:
  forall x ys, x ∘ ⋃ ys == ⋃ (fun n => x ∘ ys n)
Rels_concat_indexed_union_distr_r:
  forall xs y, ⋃ xs ∘ y == ⋃ (fun n => xs n ∘ y)
```

- 常用定理列表 4（空集与全集）：

```
Sets_union_empty_l: forall x, ∅ ∪ x == x
Sets_union_empty_r: forall x, x ∪ ∅ == x
Sets_intersect_empty_l: forall x, ∅ ∩ x == ∅
Sets_intersect_empty_r: forall x, x ∩ ∅ == ∅
Sets_union_full_l: forall x, Sets.full ∪ x == Sets.full
Sets_union_full_r: forall x, x ∪ Sets.full == Sets.full
Sets_intersect_full_l: forall x, Sets.full ∩ x == x
Sets_intersect_full_r: forall x, x ∩ Sets.full == x
Sets_equiv_empty_fact:
  forall x, x ⊆ ∅ <-> x == ∅
Sets_equiv_full_fact:
  forall x, Sets.full ⊆ x <-> x == Sets.full
```

- 常用定理列表 5（补集）：

```
Sets_intersect_complement_self
  forall x, x ∩ Sets.complement x == ∅
Sets_complement_self_intersect
  forall x, Sets.complement x ∩ x == ∅
Sets_union_complement_self:
  forall x, x ∪ Sets.complement x == Sets.full
Sets_complement_self_union
  forall x, Sets.complement x ∪ x == Sets.full
Sets_complement_complement
  forall x, Sets.complement (Sets.complement x) == x
Sets_complement_union
  forall x y,
    Sets.complement (x ∪ y) ==
    Sets.complement x ∩ Sets.complement y
Sets_complement_intersect
  forall x y,
    Sets.complement (x ∩ y) ==
    Sets.complement x ∪ Sets.complement y
```