CS257 Linear and Convex Optimization

Homework 1

Due: September 21

September 14, 2020

1. Let $f(\boldsymbol{x}) = f(x_1, x_2, x_3) = x_1^2 + e^{x_2} + e^{-x_2} + 2x_3^4$.

(a). Does f have a minimum and a maximum over the set $D = \{\boldsymbol{x} : x_1^2 + 2x_2^2 + 3x_3^2 \leq 1\}$?

(b). Does f have a minimum and a maximum on $\mathbb{R}^3$?

Hint: You don't need to solve the optimization problems to answer these questions!

2. Let $f(\boldsymbol{x}) = x_1^2 + x_1x_2 + 2x_2^2 - 3x_1 - 5x_2$.

(a). Is $f(x)$ coercive? Hint: use $x_1x_2 \geq -(x_1^2 + x_2^2)/2$.

(b). Find the minimum and maximum of $f(\boldsymbol{x})$ over $\mathbb{R}^2$ if they exist.

3. For what value of α is the following matrix positive definite?

$$A = \begin{pmatrix} 1 & \alpha & -1 \\ \alpha & 4 & 2 \\ -1 & 2 & 4 \end{pmatrix}$$

4. Are the following matrices positive definite, positive semidefinite, negative definite, negative semidefinite, or indefinite?

(a). Solve the following by writing out the eigenvalues. You can use **numpy.linalg.eigvals** to do it.

$$A = \begin{pmatrix} 7 & 2 & 0 \\ 2 & 4 & 1 \\ 0 & 1 & 3 \end{pmatrix},$$

(b). Solve the following by writing out the eigenvalues. Please calculate by hand.

$$B = \begin{pmatrix} 3 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

(c). Solve the following two by checking the principal minors.

$$C = \begin{pmatrix} -5 & 2 & 2 \\ 2 & -6 & 0 \\ 2 & 0 & -4 \end{pmatrix}, \quad D = \begin{pmatrix} -1 & 1 & -2 \\ 1 & -1 & 1 \\ -2 & 1 & 1 \end{pmatrix}$$