CS257 Linear and Convex Optimization
Homework 11

Due: November 30, 2020

December 7, 2020

1. Consider the following problem,

$$\begin{aligned}
\min_{\boldsymbol{x}\in\mathbb{R}^2} \quad & f(\boldsymbol{x}) = x_1^2 + x_2^2 \\
\text{s.t.} \quad & h_1(\boldsymbol{x}) = (x_1-1)^2 + (x_2-1)^2 - 1 = 0 \\
& h_2(\boldsymbol{x}) = (x_1-1)^2 + (x_2+\frac{1}{2})^2 - 1 = 0
\end{aligned}$$

(a). Write down the Lagrange condition.

(b). Find the points satisfying the Lagrange condition and the corresponding Lagrange multipliers.

(c). Which point is the optimal solution to the minimization problem? (Hint: A plot may be helpful.)

2. Consider the following problem that replaces the equality constraints in Problem 1 with inequality constraints,

$$\begin{aligned}
\min_{\boldsymbol{x}\in\mathbb{R}^2} \quad & f(\boldsymbol{x}) = x_1^2 + x_2^2 \\
\text{s.t.} \quad & g_1(\boldsymbol{x}) = (x_1-1)^2 + (x_2-1)^2 - 1 \le 0 \\
& g_2(\boldsymbol{x}) = (x_1-1)^2 + (x_2+\frac{1}{2})^2 - 1 \le 0
\end{aligned}$$

(a). Write down the KKT conditions.

(b). Do the optimal point you find in Problem 1(c) and its corresponding Lagrange multipliers satisfy the KKT conditions in part (a)? What can you conclude about the optimal solution to the inequality constrained problem?

3. Consider the following problem

$$\begin{aligned}
\min \quad & (x_1-\frac{9}{4})^2 + (x_2-2)^2 \\
\text{s.t.} \quad & -x_1^2 + x_2 \ge 0 \\
& x_1 + x_2 \le 6 \\
& x_1, x_2 \ge 0
\end{aligned}$$

For each of the following points, determine whether it is an optimal solution to the above problem and show you arguments,

$$\boldsymbol{x}^{(1)} = \begin{bmatrix} \frac{9}{4} \\ 2 \end{bmatrix}, \boldsymbol{x}^{(2)} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \boldsymbol{x}^{(3)} = \begin{bmatrix} \frac{3}{2} \\ \frac{9}{4} \end{bmatrix}$$

(Hint: Try if you can find Lagrange multipliers satisfying the KKT conditions. Note you can easily check which constraints are active.)