CS257 Linear and Convex Optimization

Homework 12

Due: December 14, 2020

December 7, 2020

1. Consider the following problem,

$$\begin{aligned} \min_{\boldsymbol{x}\in\mathbb{R}^2} \quad & f(\boldsymbol{x}) = x_1^2 + x_2^2 \\ \text{s.t.} \quad & g_1(\boldsymbol{x}) = (x_1-1)^2 + (x_2-1)^2 - 1 \leq 0 \\ & g_2(\boldsymbol{x}) = (x_1-1)^2 + (x_2+\frac{1}{2})^2 - 1 \leq 0 \end{aligned}$$

Write down the KKT conditions and find the optimal point $\boldsymbol{x}^*$ and the corresponding Lagrange multipliers.

2. Consider the following problem,

$$\begin{aligned} \min_{\boldsymbol{x}\in\mathbb{R}^2} \quad & f(\boldsymbol{x}) = (x_1-1)^2 + (x_2+1)^2 \\ \text{s.t.} \quad & g(\boldsymbol{x}) = x_1 - x_2 + 1 \leq 0 \end{aligned}$$

(a). Write down the KKT conditions and find the optimal point, the corresponding Lagrange multiplier and the optimal value.

(b). Find the Lagrange dual function. Note that we do not require $\mu \geq 0$ in the dual function.

(c). Derive the Lagrange dual problem.

(d). Write down the KKT conditions for the Lagrange dual problem. Find the optimal point, the corresponding Lagrange multiplier and the optimal value.