EI331 Signals and Systems
Homework 10

Due: Thursday, May 23

May 18, 2019

1. (C2.2) For each of the following functions, determine the sets of points at which the function is differentiable and analytic, respectively.

(a). $f(z) = x^2 - jy$

(b). $f(z) = 2x^3 + j3y^3$

(c). $f(z) = xy^2 + jx^2y$

(d). $f(z) = \sin x \cosh y + j \cos x \sinh y$

2. (C2.3) For each of the following functions, determine the set of points at which the function is analytic.

(a). $f(z) = (z-1)^5$

(b). $f(z) = z^3 + 2jz$

(c). $f(z) = \frac{1}{z^2-1}$

(d). $f(z) = \frac{az+b}{cz+d}$, where $|c| + |d| \neq 0$

3. (C2.10) Assume f is analytic on a domain D. Show that f is constant if any of the following conditions holds.

(a). $f' = 0$ on D

(b). f is real

(c). $\bar{f}$ is analytic on D

(d). $|f|$ is constant on D

(e). $\arg f$ is constant on D

(f). $au + bv = c$, where the constants $a, b, c \in \mathbb{R}$ are not all zero.

4. (C2.12) Find all the roots of $\sin z + \cos z = 0$

5. Evaluate $\mathrm{Log}(-1 + j\sqrt{3})$, and find its principal value.

6. (C2.18) Evaluate 3^j and $(1+j)^j$.

7. Evaluate $\int_0^{1+j}(x^2+jy)dz$ along $y=x$ and $y=x^2$

8. Evaluate the integral $\int_\gamma \frac{\bar{z}}{|z|}dz$, where γ is a positively oriented circle of radius R centered at $z=0$.

9. (C3.7.7) Evaluate the integral $\int_\gamma \frac{dz}{(z^2+1)(z^2+4)}$, where γ is a positively oriented circle of radius $\frac{3}{2}$ centered at $z=0$.

10. (C3.12) Let $D=\{z : \operatorname{Re} z>0\}$ be the right half plane and z_0 a point in D with $|z|=1$. Let γ be a piecewise smooth curve in D that connects $z=0$ and z_0. Show $\operatorname{Re}\int_\gamma \frac{1}{1+z^2}dz=\frac{\pi}{4}$. Hint: Argue that you can replace γ by γ_1 and γ_2, where γ_1 be the line segment connecting $z=0$ and $z=1$ and γ_2 the arc on the unit circle connecting $z=1$ and z_0.

11. (C3.17) Let f and g be two analytic functions on a domain D and γ a piecewise smooth Jordan curve in D whose interior D_1 lies in D. If $f(z)=g(z)$ on γ, then $f(z)=g(z)$ on D_1.

Remark. This shows that the values of an analytic function on a domain are completely determined by its values on the boundary.