EI331 Signals and Systems
Homework 11

Due: Thursday, May 30

May 23, 2019

1. Evaluate the following integrals. The circles are positively oriented.

(a). (C3.7) $\displaystyle\int_{|z|=2} \frac{\sin z dz}{\left(z-\frac{\pi}{2}\right)^2}$

(b). (C3.9) $\displaystyle\int_{|z|=1} \frac{e^z dz}{(z-\alpha)^3}$, where $\alpha \neq 1$

2. (C3.16) f is analytic on the annulus $0 < |z| < 1$ and $\int_C f(z)dz = 0$ for any circle $C : |z| = r$, where $0 < r < 1$. Is $f(z)$ necessarily analytic at $z = 0$? If your answer is yes, prove it. If your answer is no, give a counterexample.

3. (C3.21) Let f be analytic on a domain D and C a positively oriented piecewise smooth Jordan curve in D whose interior is also in D. For any point $z_0 \in D \setminus C$, show

$$\int_C \frac{f'(z)}{z-z_0} dz = \int_C \frac{f(z)}{(z-z_0)^2} dz$$

4. (C4.9) If $\sum_{n=0}^{\infty} c_n$ converges but $\sum_{n=0}^{\infty} |c_n|$ diverges, show that the series $\sum_{n=0}^{\infty} c_n z^n$ has radius of convergence $R = 1$.

5. (C4.11) Find the Taylor series expansions at $z = 0$ and the radius of convergence for each of the following functions.

(a). $\displaystyle\frac{1}{(1+z^2)^2}$

(b). $\sinh z$

(c). $\cosh z$

(d). $e^{z^2} \sin z^2$

6. (C4.12) Find the Taylor series at z_0 and the radius of convergence for each of the following functions.

(a). $\displaystyle\frac{z-1}{z+1}$, $z_0 = 1$

(b). $\dfrac{z}{(z+1)(z+2)}$, $z_0 = 2$

(c). $\dfrac{1}{z^2}$, $z_0 = -1$

(d). $\dfrac{1}{4-3z}$, $z_0 = 1 + j$

7. (C4.16) Find the Laurent series in the given annuli for each of the following functions.

(a). $\dfrac{1}{(z^2+1)(z-2)}$, $1 < |z| < 2$

(b). $\dfrac{1}{z(1-z)^2}$, $0 < |z| < 1$; $0 < |z-1| < 1$

(c). $\dfrac{1}{(z-1)(z-2)}$, $0 < |z-1| < 1$; $1 < |z-2| < \infty$

(d). $\dfrac{1}{z^2(z-j)}$, $0 < |z-j| < 1$; $1 < |z-j| < \infty$

(e). $\dfrac{(z-1)(z-2)}{(z-3)(z-4)}$, $3 < |z| < 4$; $4 < |z| < \infty$

8. (C4.20) Evaluate $\displaystyle\int_C \left(\sum_{n=-2}^{\infty} z^n \right) dz$, where C is a piecewise smooth Jordan curve in the open disk $B(0,1)$ that does not go through the origin.