EI331 Signals and Systems

Homework 12

Due: Thursday, June 6

May 31, 2019

1. (C5.8) For each of the following functions, compute the residues at its finite singularities.

(a). $f(z) = \dfrac{z+1}{z^2-2z}$

(b). $f(z) = \dfrac{1-e^{2z}}{z^4}$

(c). $f(z) = \dfrac{1+z^4}{(z^2+1)^3}$

(d). $f(z) = \dfrac{z}{\cos z}$

(e). $(z) = \cos\dfrac{1}{1-z}$

2. (C5.9) Evaluate the following integrals, where all circles are positively oriented.

(a). $\displaystyle\int_{|z|=\frac{3}{2}} \frac{\sin z}{z} dz$

(b). $\displaystyle\int_{|z|=2} \frac{e^{2z}}{(z-1)^2} dz$

(c). $\displaystyle\int_{|z|=\frac{3}{2}} \frac{1-\cos z}{z^m} dz$, $m \in \mathbb{Z}$

3. (C5.11) For each of the following functions, find the residues at ∞

(a). $f(z) = \dfrac{e^z}{z^2-1}$

(b). $f(z) = \dfrac{1}{z(z+1)^4(z-4)}$

4. (C5.12) Evaluate the following integrals, where all circles are positively oriented.

(a). $\displaystyle\int_{|z|=3} \frac{z^{15}}{(z^2+1)^2(z^4+2)^3} dz$

(b). $\displaystyle\int_{|z|=r} \frac{z^{2n}}{z^n+1} dz$, where n is a positive integer, and $r > 1$.

5. (C5.13) Evaluate the following integrals.

(a). $\int_0^{2\pi} \frac{1}{5+3\sin\theta} d\theta$

(b). $\int_0^{\infty} \frac{x^2}{1+x^4} dx$

(c). $\int_{-\infty}^{\infty} \frac{\cos x}{x^2+4x+5} dx$

6. (OWN 10.21) For each of the following signals, find its z-transform and the ROC. Point out the finite zeros and poles, their orders, and whether the DTFT exists.

(a). $\delta[n+5]$

(b). $\delta[n-5]$

(c). $(-1)^n u[n]$

(d). $(\frac{1}{2})^{n+1} u[n+3]$

(e). $(-\frac{1}{3})^n u[-n-2]$

(f). $(\frac{1}{4})^n u[3-n]$

(g). $2^n u[-n] + (\frac{1}{4})^n u[n-1]$

(h). $(\frac{1}{3})^{n-2} u[n-2]$

7. (OWN 10.23) Find the inverse z transforms.

(a). $X(z) = \frac{1-z^{-1}}{1-\frac{1}{4}z^{-2}}$, $|z| > \frac{1}{2}$

(b). $X(z) = \frac{1-z^{-1}}{1-\frac{1}{4}z^{-2}}$, $|z| < \frac{1}{2}$

(c). $X(z) = \frac{z^{-1}-\frac{1}{2}}{1-\frac{1}{2}z^{-1}}$, $|z| > \frac{1}{2}$

(d). $X(z) = \frac{z^{-1}-\frac{1}{2}}{1-\frac{1}{2}z^{-1}}$, $|z| < \frac{1}{2}$

(e). $X(z) = \frac{z^{-1}-\frac{1}{2}}{(1-\frac{1}{2}z^{-1})^2}$, $|z| > \frac{1}{2}$

(f). $X(z) = \frac{z^{-1}-\frac{1}{2}}{(1-\frac{1}{2}z^{-1})^2}$, $|z| < \frac{1}{2}$