EI331 Signals and Systems
Homework 13

Will not be collected or graded

June 8, 2019

1. (OWN 10.34) A causal LTI system is described by the following difference equation

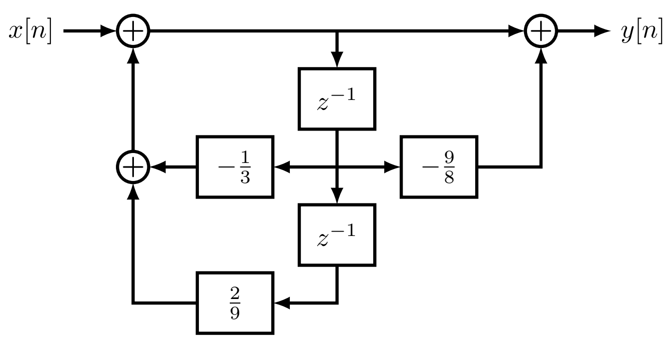
$$y[n] = y[n-1] + y[n-2] + x[n-1]$$

(a). Find the system function $H(z)$. Plot the finite poles and zeros of $H(z)$ and indicate the ROC.

(b). Find the impulse response of the system.

(c). Find the impulse response of a stable (noncausal) system described by the difference equation.

2. (OWN 10.37) Consider a causal LTI system whose input $x[n]$ and output $y[n]$ are related by the following block diagram representation.



(a). Determine a difference equation relating $y[n]$ and $x[n]$

(b). Is the system stable?

3. (OWN 10.42a) Use the unilateral z-transform to find the zero-input and zero-sate responses for $n \geq 0$ of the causal system described by the following difference equation with the given initial condition.

$$y[n] + 3y[n-1] = x[n]$$

$$x[n] = (\frac{1}{2})^n u[n]$$

$$y[-1] = 1$$

4. (OWN 10.46) A sequence $x[n]$ is the output of an LTI system whose input is $s[n]$. The system is described by the difference equation

$$x[n] = s[n] - e^{8a}s[n-8]$$

where $0 < a < 1$.

(a). Find the system function

$$H_1(z) = \frac{X(z)}{S(z)}$$

and plot its poles and zeros in the z-plane. Indicate the ROC.

(b). We wish to recover $s[n]$ from $x[n]$ with an LTI system. Find the system function

$$H_2(z) = \frac{Y(z)}{X(z)}$$

such that $y[n] = s[n]$. Find all possible ROCs for $H_2(z)$, and for each, tell whether or not the system is causal or stable.

(c). Find all possible choices for the unit impulse response $h_2[n]$ such that

$$y = h_2 * x = s$$

5. Consider a system whose response y to an input x is given by the solution to the following initial value problem,

$$y[n] + a_1 y[n-1] + a_2 y[n-2] = \sum_{k=0}^{M} b_k x[n-k]$$

with initial conditions $y[-1] = c_1$ and $y[-2] = c_2$. When the input is $x_1[n] = 2^n u[n]$, the response is

$$y_1[n] = \frac{2}{3}(-1)^n - (-2)^n + \frac{1}{3}2^n, \quad n \geq 0$$

When the input is $x_2[n] = u[n]$, the response is

$$y_2[n] = \frac{1}{2}(-1)^n - \frac{2}{3}(-2)^n + \frac{1}{6}, \quad n \geq 0$$

Find the system function $H(z)$ of the causal LTI system described by the same difference equation, its impulse response, the difference equation and its initial conditions $y[-1]$ and $y[-2]$.

6. (OWN 9.21) Find the Laplace transform and the associated ROC for the following signals. Find the finite poles and zeros and their orders.

(a). $x(t) = e^{-2t}u(t) + e^{-3t}u(t)$

(b). $x(t) = e^{2t}u(-t) + e^{3t}u(-t)$

(c). $x(t) = te^{-2|t|}$

(d). $x(t) = |t|e^{-2|t|}$

(e). $x(t) = \begin{cases} 1, & 0 \le t \le 1 \\ 0, & \text{elsewhere} \end{cases}$

(f). $x(t) = \delta(t) + u(t)$

7. (OWN 9.22) Find the signals with the following Laplace transforms in the given ROC.

(a). $\dfrac{s+1}{(s+1)^2+9}, \quad \operatorname{Re} s < -1$

(b). $\dfrac{s+2}{s^2+7s+12}, \quad -4 < \operatorname{Re} s < -3$

(c). $\dfrac{(s+1)^2}{s^2-s+1}, \quad \operatorname{Re} s > \frac{1}{2}$

(d). $\dfrac{s^2-s+1}{(s+1)^2}, \quad \operatorname{Re} s > -1$

8. (OWN 9.31) Consider an LTI system whose input and output are related by the ODE

$$\frac{d^2}{dt^2}y(t) - \frac{d}{dt}y(t) - 2y(t) = x(t)$$

(a). Find the system function $H(s)$ and its finite zeros and poles.

(b). Find the impulse response $h(t)$ in the each of the following cases

1. The system is stable
2. The system is causal
3. The system is neither stable nor causal

9. (OWN 9.32) A causal LTI system with impulse response $h(t)$ has the following properties,

(a). When the input to the system is $x(t) = e^{2t}$ for all t, the output is $y(t) = \frac{1}{6}e^{2t}$ for all t.

(b). The impulse response $h(t)$ satisfies the following ODE,

$$\frac{d}{dt}h(t) + 2h(t) = e^{-4t}u(t) + bu(t)$$

where b is an unknown constant.

Determine the system function $H(s)$. Find its finite zeros and poles.

10. (OWN 9.37) Draw a direct form representation for the causal LTI systems with the following system function

$$H(s) = \frac{s^2-5s+6}{s^2+7s+10}$$

11. Use the unilateral Laplace transform to find the zero-input and zero-state responses for $t > 0$ of the causal system described the following ODE

$$\frac{d}{dt}y(t) + 2y(t) = \frac{d}{dt}x(t) + x(t)$$

$$x(t) = e^{-2t}u(t)$$

$$y(0_-) = 2$$