EI331 Signals and Systems
Homework 5

Due: Thursday, April 4

March 29, 2019

1. (OWN 3.3) Find the fundamental frequency ω_0 of the following CT signal

$$x(t) = 2 + \cos\left(\frac{2\pi}{3}t\right) + 4\sin\left(\frac{5\pi}{3}t\right)$$

and its Fourier coefficients in the complex exponential form, i.e. $\hat{x}[k]$ such that

$$x(t) = \sum_{k=-\infty}^{\infty} \hat{x}[k] e^{j\omega_0 t}$$

2. (OWN 3.4)

(a). Use Fourier analysis equation $\hat{x}[k] = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt$ to calculate the Fourier coefficients $\hat{x}[k]$ of the following CT periodic signal

$$x(t) = \begin{cases} 1.5, & 0 \le t \le 1 \\ -1.5 & 1 \le t \le 2 \end{cases}$$

with fundamental frequency $\omega_0 = \pi$.

(b). Find the sum of the following series

$$\sum_{k=1}^{\infty} \frac{1}{(2k-1)^2}$$

3. (OWN 3.5) Let x_1 be a CT periodic signal with fundamental frequency ω_1. Find the Fourier coefficients $\hat{x}_2$ of the following CT signal

$$x_2(t) = x_1(1-t) + x_1(t-1)$$

in terms of the Fourier coefficients $\hat{x}_1$ of x_1.

4. (OWN 3.17) Consider three CT systems S_1, S_2, S_3 whose responses y_1, y_2, y_3 to the input $x(t) = e^{j5t}$ are given by

$$y_1(t) = te^{j5t}$$
$$y_2(t) = e^{j5(t-1)}$$
$$y_3(t) = \cos(5t)$$

Which of those systems **cannot** be LTI? Justify your answer.

5. (OWN 3.26) Let $x(t)$ be a periodic signals with the following Fourier coefficients

$$\hat{x}[k] = \begin{cases} 2, & k = 0 \\ j(\frac{1}{2})^{|k|}, & \text{otherwise} \end{cases}$$

Use Fourier series properties to answer the following questions.

(a). Is $x(t)$ real?

(b). Is $x(t)$ even?

(c). Is $dx(t)/dt$ even?

6. (OWN 3.23) In each of the following cases, we specify the Fourier coefficients of a CT signal with period 4. Find the signal $x(t)$ in each case.

(a). $\hat{x}[k] = \begin{cases} 0, & k = 0 \\ (j)^k \frac{\sin(k\pi/4)}{k\pi}, & \text{otherwise} \end{cases}$

Hint: Use the Fourier series of periodic square wave.

(b). $\hat{x}[k] = \begin{cases} jk, & |k| < 3 \\ 0, & \text{otherwise} \end{cases}$

Express x in terms of real-valued functions.

7. (OWN 3.8) Suppose we are given the following information about a CT signal $x(t)$

(a). x is real and odd

(b). x is periodic with period $T = 2$

(c). its Fourier coefficients $\hat{x}[k] = 0$ for $|k| > 1$

(d). $\frac{1}{2}\int_0^2 |x(t)|^2 dt = 1$

Specify two different signals that satisfy these conditions.

8. (OWN 3.61) A function x is an eigenfunction of an LTI system T if it is not identically zero and $T(x) = \lambda x$ for some constant $\lambda \in \mathbb{C}$. We have seen that exponentials are eigenfunctions of LTI systems. This problem shows that some LTI systems may have other eigenfunctions, but exponentials are the only eigenfunctions of **all** LTI systems.

(a). Consider any signal x continuous at $t = 0$. What's the response to x of the LTI system with impulse response $h(t) = \delta(t)$? Is x an eigenfunction of the system? What's the corresponding eigenvalue?

(b). Consider an LTI system whose impulse response h is real and even. Show that $\cos(\omega t)$ is an eigenfunction of this system with eigenvalue $H(j\omega)$, where $H(j\omega) = \int_{\mathbb{R}} h(t)e^{-j\omega t}dt$.

(c). Consider the LTI system with impulse response $h(t) = u(t)$. Suppose $\phi(t)$ is an eigenfunction of this system with eigenvalue λ. Find the differential equation that $\phi(t)$ must satisfy and solve it to find $\phi(t)$. This shows that ϕ must be an exponential.

9. The first two Legendre polynomials are

$$P_0(t) = 1, \quad P_1(t) = t$$

(a). Show that P_0 and P_1 are orthogonal in $L_2[-1,1]$, the space of functions on $[-1,1]$ with finite 2-norm, where the inner product is defined by

$$\langle f, g \rangle = \int_{-1}^{1} f(t)\overline{g(t)}dt$$

Are they unit vectors?

(b). Find the best mean-square approximation to t^2 on $[-1,1]$ by a linear function, i.e. solve the following minimization problem,

$$\min_{a,b\in\mathbb{C}} \int_{-1}^{1} |t^2 - a - bt|^2 dt$$