# EI331 Signals and Systems
# Homework 8

Due: Tuesday, April 23
Late submission not accpeted

April 18, 2019

**1.** (OWN 4.21) Find the Fourier transforms of the following signals. You can use any results in the textbook or that we have derived in class. It is easiest to use properties of Fourier transforms.

(a). $\sum_{k=0}^{\infty} a^k \delta(t - kT)$, $|a| < 1$

(b). $x$ as shown in Figure 1.

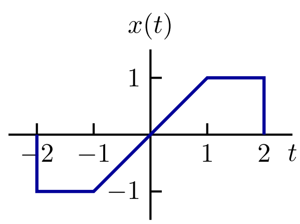



**Figure 1: Problem 1**

**2.** (OWN 4.22) Find the CT signal with the Fourier transform shown in Figure 2.

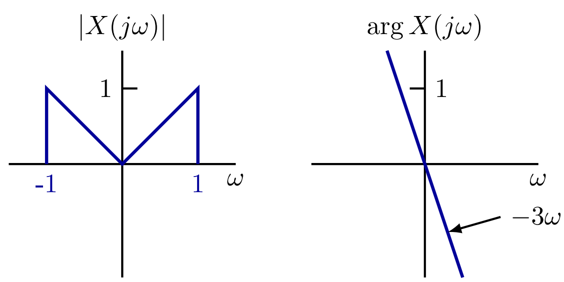



**Figure 2: Problem 2**

**3.** (OWN 4.25) Let $X(j\omega)$ denote the Fourier transform of the signal $x(t)$ in Figure 3.

(a). Find $\arg X(j\omega)$

(b). Find $X(j0)$

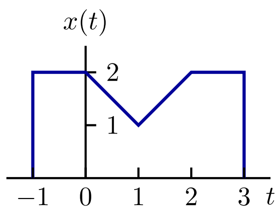



**Figure 3: Problem 3**

(c). Find $\int_{\mathbb{R}} X(j\omega)d\omega$

(d). Evaluate $\int_{\mathbb{R}} X(j\omega)\frac{2\sin\omega}{\omega}e^{j2\omega}d\omega$

(e). Evaluate $\int_{\mathbb{R}} |X(j\omega)|^2 d\omega$

(f). Sketch the inverse Fourier transform of $\mathrm{Re}X(j\omega)$

Note: Do not explicitly evaluate $X(j\omega)$.

**4.** (OWN 4.33) Consider the LTI system described by the following ODE

$$y'' + 6y' + 8y = 2x$$

(a). Find its frequency response

(b). Find its impulse response

(c). Find the response to $x(t) = te^{-2t}u(t)$ using the Fourier transform method.

**5.** (OWN 6.22) Consider the **lowpass differentiator** with frequency response $H(j\omega)$ in Figure 4(a).

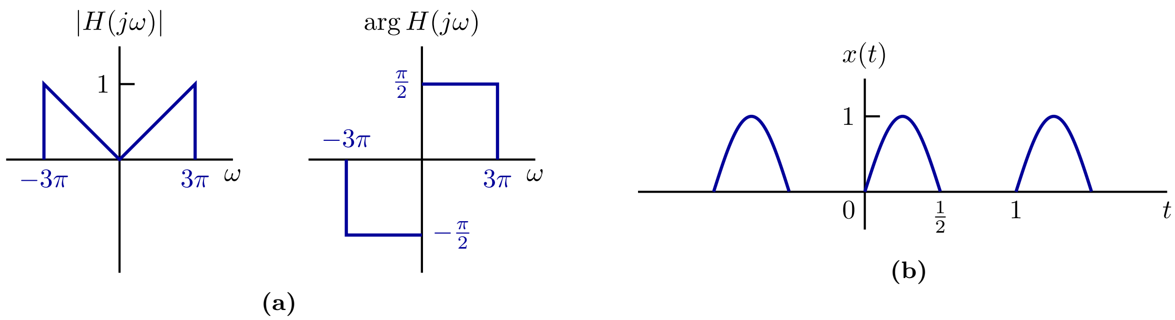



**Figure 4: Problem 5**

For each of the following inputs $x(t)$, determine the filtered output $y(t)$.

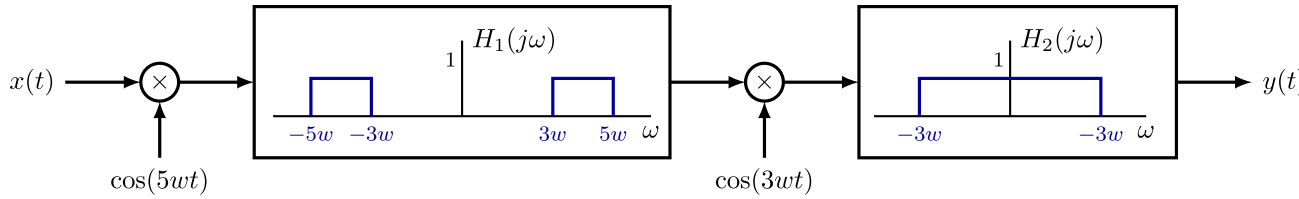
(a). $x(t) = \cos(2\pi t + \theta)$

(b). $x(t) = \cos(4\pi t + \theta)$

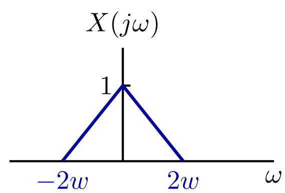
(c). $x(t)$ is a **half-wave rectified** sine wave in Figure 4(b), which is given by

$$x(t) = \begin{cases} \sin(2\pi t), & m \le t \le (m+1/2) \\ 0, & (m+1/2) \le t \le m \end{cases}, \quad m \in \mathbb{Z}$$

**6.** (OWN 8.22) Consider the system in Figure 5(a). For the input signal with the spectrum in Figure 5(b), determine and sketch the spectrum $Y(j\omega)$ of $y$.




**(a)**




**(b)**

**Figure 5: Problem 6**