EI331 Signals and Systems
Homework 9

Due: Thursday, May 16

May 11, 2019

1. Find the DTFT of the following signals, where $|a| < 1$

(a). $x_1[n] = a^n u[n-1]$

(b). $x_2[n] = a^{|n-2|}$

(c). $x_3[n] = a^n \sin(\omega_0 n) u[n]$

(d). $x_4[n] = \sum_{k=0}^{\infty} a^n \delta[n-3k]$

2. Find the DT signals corresponding to the following DTFTs.

(a). (OWN 5.4(a)) $X_1(e^{j\omega}) = \sum_{k=-\infty}^{\infty} [2\pi\delta(\omega - 2\pi k) + \pi\delta(\omega - \frac{\pi}{2} - 2\pi k) + \pi\delta(\omega + \frac{\pi}{2} - 2\pi k)]$

(b). (OWN 5.4(b)) $X_2(e^{j\omega}) = \begin{cases} 2j, & 0 < \omega \le \pi \\ -2j, & -\pi < \omega \le 0 \end{cases}$

(c). $X_3(e^{j\omega}) = \cos^2(\omega)$

3. (OWN 5.8) Find the signal with the following DTFT,

$$X(e^{j\omega}) = \frac{1}{1 - e^{-j\omega}} \frac{\sin(\frac{3}{2}\omega)}{\sin\frac{\omega}{2}} + 5\pi\delta(\omega), \quad -\pi < \omega \le \pi$$

4. (OWN 5.9) Find the real DT signal $x[n]$ whose DTFT $X(e^{j\omega})$ has the following properties

(a). $x[n] = 0$ for $n > 0$

(b). $x[0] > 0$

(c). $\text{Im } X(e^{j\omega}) = \sin\omega - \sin 2\omega$

(d). $\frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = 3$

5. Consider a DT LTI system described by the following difference equation

$$y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = x[n] + \frac{1}{3}x[n-1]$$

(a). Find the frequency response of the system

(b). Find the impulse response of the system

(c). Find the response to the input $x[n] = \left(\frac{1}{2}\right)^n u[n]$ using the Fourier transform method.

6. Given that the Nyquist rate of a CT signal $x(t)$ is ω_0, for each of the following signals, determine the range of sampling rates that guarantee there is no aliasing,

(a). $x(t) + x(t-1)$

(b). $\frac{dx(t)}{dt}$

(c). $x^2(t)$

(d). $x(t)\cos(\omega_0 t)$

7. (OWN 7.26) The sampling theorem we discussed in class states that a signal should be sampled at a rate greater than twice its highest frequency. However, bandpass signals can be sampled at a lower rate. Consider a bandpass signal $x(t)$ with spectrum $X(j\omega) \neq 0$ only for $\omega_1 \leq |\omega| < \omega_2$, where $\omega_1 > \omega_2 - \omega_1$. Let $p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$ and $x_p(t) = x(t)p(t)$. Let

$$H_{bp}(j\omega) = \begin{cases} A, & \text{if } 0 < \omega_a \leq |\omega| \leq \omega_b \\ 0, & \text{otherwise} \end{cases}$$

Find the maximum value of T and the values of A, ω_a, ω_b such that $x(t)$ can be reconstructed by sending $x_p(t)$ through the filter $H_{bp}(j\omega)$. (see Figure P7.26 of the textbook)

8. (C1.26) Find the image of the following curves under the mapping $w = 1/z$.

(a). $|z| = 2$

(b). $\operatorname{Re} z = \operatorname{Im} z$

(c). $\operatorname{Re} z = 1$

(d). $|z - 1| = 1$

9. (C1.31) Let

$$f(z) = \frac{1}{2j}\left(\frac{z}{z^*} - \frac{z^*}{z}\right), \ z \neq 0$$

where z^* is the complex conjugate of z. Show that $\lim_{z \to 0} f(z)$ does not exist.