Review: Homeworks

Hongfei Fu

John Hopcroft Center for Computer Science
Shanghai Jiao Tong University

May. 24th

Problem 1: (20' in the exam)

Sample Homework

Let X, Y be locations (i.e., program variables). Let the state σ be given by $\sigma(X) = 3$ and $\sigma(Y) = 5$. Solve the following problems through derivation trees.

(a) For $a = X - 1$, determine the integer n such that $\langle a, \sigma \rangle \rightarrow n$.

(b) For $b = Y - X \leq 2$, determine the truth value t such that $\langle b, \sigma \rangle \rightarrow t$.

(c) For $c =$ **if** $Y - X \leq 2$ **then** $Y := X - 1$ **else skip** , determine the state σ' such that $\langle c, \sigma \rangle \rightarrow \sigma'$.

Problem 2: (20' in the exam)

Sample Homework

Consider the command

$$c = \textbf{while } X \leq 10 \textbf{ do } X := X + 1$$

where X is a location (program variable). For each initial state σ, determine the state σ' such that $\langle c, \sigma \rangle \rightarrow \sigma'$ and verify your answer.

Problem 3: (13' in the exam)

Sample Homework

- $w :=$ **while** $X \geq 0$ **do** $X := X + 1$

Prove that $\forall \sigma, \sigma'. (\sigma(X) \geq 0 \Rightarrow \langle w, \sigma \rangle \not\to \sigma')$.

Problem 4: (13' in the exam)

Sample Homework 1

Let D be a non-empty set and $(D \rightharpoonup D)$ be the set of all partial functions from D to D. Prove that the partial order $((D \rightharpoonup D), \subseteq)$ (i.e., the set of partial functions ordered by set inclusion) is a complete partial order with bottom.

Sample Homework 2

- D, E, F: cpo's (with their implicit ordering relations)
- $f : D \to E$ and $g : E \to F$: continuous functions

Prove that the function $g \circ f : D \to F$ is continuous.

Problem 5 (14' in the exam)

Sample Homework 1

Consider the command c to be $Z := X; X := Y; Y := Z$ with locations X, Y, Z. Prove through Hoare rules that the partial correctness assertion

$$\{X = i \wedge Y = j\}c\{X = j \wedge Y = i\}$$

is valid, where i, j are integer variables.

Sample Homework 2

Let c be the command **while** $X \leq 100$ **do** $X := X + 2$ with location X. Prove through the Hoare rules that

$$\models \{X \leq 100 \wedge (\exists i.X = 2 \times i + 1)\}c\{X = 101\}$$

where i is an integer variable.

Problem 6 (20' in the exam)

Sample Homework

- x : **int**
- y : **int** $\rightarrow$ **int**
- $fact := \textbf{rec}y.(\lambda x.(\textbf{if } x \textbf{ then } 1 \textbf{ else } x \times (y\ (x-1))))$
- Find the type of *fact* through the typing rules.
- Evaluate $(fact\ 2)$ under the eager operational semantics.
- Give the evaluation result for $(fact\ n)$ $(n \in \mathbb{N})$ and prove your answer.

About the Exam

Information

- Time: June 14th, 8:55 – 11:40
  (Friday, the 16th week, the usual time when we have classes)
- Place: Room 111, Chen Rui Qiu Building
  (the room where we have classes)

Principle

- Write out every essential point (e.g. well-foundedness, consequence rule, etc.).
- You don't need to write out the points in detail.
- The materials on rules for the operational semantics of the imperative language in Chapter 2, the Hoare rules, the typing and eager evaluation rules for the functional language in Chapter 11 will be provided (you don't need to recite them).