Leveraging Generators with Complementary Capabilities for Robust Multi-stage Power Grid Operations

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***Abstract*—In this paper, we develop online multi-stage decisions for procuring and dispatching generators with diverse capabilities to provably assure reliability of power grid operations under high renewable uncertainty. We jointly consider both the day-ahead reliability assessment commitment (RAC) and the real-time dispatch problems. We first focus on the real-time dispatch problem and define "maximally robust algorithms," in the sense that they are not dominated by any other algorithm in terms of reliability. We characterize a class of maximally robust algorithms using the concept of "safe dispatch set," which also provides conditions for verifying grid reliability for RAC. However, in general such safe dispatch sets may be difficult to compute. We then develop efficient computational algorithms for characterizing the safe dispatch sets. Specifically, for a simpler single-bus two-generator case, we show that the safe dispatch sets can be exactly characterized by a polynomial number of convex constraints. Then, based on this two-generator characterization, we develop a new solution for the multi-bus multi-generator case using the idea of *virtual demand splitting* (VDS), which can effectively compute a suitable subset of the safe dispatch set. A distinct advantage of our proposed approach is that it intelligently exploits the complementary capabilities of generators with different ramping capabilities, and thus leads to a larger safe dispatch set than existing approaches in the literature based purely on affine policies. Our numerical results on an IEEE 30-bus system demonstrate that a VDS-based solution outperforms the standard approaches in the literature in terms of reliability, without sacrificing economy.**

I. INTRODUCTION

The high variability and uncertainty of renewable energy poses an immense challenge to the existing power grid. Note that, for power grid operations, it is considered the top priority to maintain the power supply-demand balance, otherwise the imbalance can cause the system to collapse with catastrophic consequences [1, p. 49]. Since renewable energy is highly variable, the grid operator has to prepare a sufficient amount of generation from fossil-fuel generators, often with different output levels and ramping capabilities, to compensate for the renewable uncertainty. However, these generation resources not only are costly, but also have hard physical constraints that cannot be violated at any time (so do the constraints of the transmission grid). Hence, it becomes extremely challenging to determine which set of resources needs to be procured beforehand, and how to dispatch these heterogeneous resources in real time, in order to ensure reliable grid operations with the least amount of resources.

In most parts of the US, the responsibility of maintaining grid reliability at all times is on the Independent System Operator (ISO) [2–4]. An ISO typically runs (at least) two markets. In the day-ahead market, a security-constrained unit-commitment and economic-dispatch schedule is computed for every hour of the next day, based on forecast of demand and supply. Then, in the real-time market during the following day, the ISO must adjust the dispatch decisions every 5 minutes to match any actual demand deviation. In-between the real-time market and the day-ahead market (or even multiple times during the following day), the ISO runs a RAC[1] (Reliability Assessment Commitment) stage to determine whether additional generators need to be committed. In this work, we are most interested in the decisions for both the RAC stage and the real-time dispatch stage, in order to ensure grid reliability under high penetration of renewable energy.

Many studies of this RAC and real-time dispatch problem have been based on two-stage stochastic or robust optimization [5–8], which assumes that there is a second stage where the future renewable energy supply for all time slots is known. In practice, however, renewable supply is revealed sequentially in many stages, and at each stage the decisions must be made in an online manner, without knowing the renewable supply at future stages. It has been pointed out in [9] that such a two-stage assumption will lead to decisions that are neither implementable in real-time, nor reliable for the RAC stage, i.e., it may incorrectly identify a system as reliable, even though the system is not. Correct multi-stage sequential decisions can be solved via dynamic programming [10] or adaptive robust optimization (ARO) [11], [12]. However, the solution usually incurs exponentially high complexity. To overcome this difficulty, the authors of [9] and [13] recently proposed to use AARO (Adaptive Affine Robust Optimization) to this problem, which further restrict the decision rules to the class of affine policies.

Although AARO reduces the computational complexity, we have found that it does not efficiently exploit the complementary capabilities of a fleet of generators with diverse power levels and ramp limits. As a result, with the same amount of generation resources, AARO may support a much lower level of renewable uncertainty than the new policies that we

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[1]Depending on the ISOs [4], this stage may also be called RUC (Reliability Unit Commitment)

will develop in this paper. To see the weakness of of AARO, consider the case where there are two generators: a slow generator with a large power output range but a small ramping rate, and a fast generator with a small power output range but no ramping constraints. Intuitively, in the affine policy, only a small fraction of uncertain demand can be sent to the slow generator due to its limited ramping capability. Similarly, only a small fraction of uncertainty can be sent to the fast generator due to the limited power level that it can support. Together, the level of uncertainty that can be supported by these two generators using the class of affine policies may be very small, even though a much higher level of uncertainty can be supported by the same set of resources using more intelligent policies (see Example 1 in Section II-B for more details). Thus, it remains an open problem how to develop more efficient and equally-tractable ways to deal with multi-stage decisions in such settings.

In this paper, we propose new computationally-efficient algorithms for online multi-stage decisions that can ensure grid reliability for a given level of renewable uncertainty. In contrast to [9] and [13], we aim to exploit the complementary capabilities of heterogeneous generators to support a higher level of renewable uncertainty than AARO. Towards this end, we first introduce the concept of "safe dispatch sets" (see Sec. III for the rigorous definition). We show that a class of algorithms based on safe dispatch sets is "maximally robust," i.e., algorithms of this class are not dominated by any other algorithm in terms of reliability (see Sec. III for details). However, for general settings, computing these safe dispatch sets incurs high computational complexity. Our main contribution is to develop efficient computational algorithms (in Sec. IV) for characterizing these safe dispatch sets. Towards this end, we use a divide-and-conquer approach. First, for a simpler single-bus case with a fast generator and a slow generator, we develop a precise characterization of the safe dispatch set that not only optimally exploits the complementary capabilities of the two generators, but also involves only a polynomial number of convex constraints. Second, this two-generator characterization becomes a building block towards our solution for the general multi-bus case with many generators of arbitrary power levels and ramping rates. Specifically, we form *virtual fast/slow generators* (VFG/VSG) from actual physical generators (which allows us to use the two-generator characterization), and then use the idea of *virtual demand splitting* (VDS), which can effectively compute a suitable subset of the safe dispatch set. Note that while such demand splitting can be viewed as a form of affine policies, it operates over generator pairs instead of over each individual generator. Therefore, the overall policy (VDS + two-generator characterization) more-efficiently exploit the complementary capabilities of different generators, and as a result produce a larger safe dispatch set than the AARO policies in [9] and [13]. As we illustrate in the numerical results in Sec. V, our proposed solution outperforms state-of-the-art multi-stage AARO algorithm [9] in both RAC and real-time dispatch. Compared to the current practice of using standard ED algorithm for real-time dispatch, our proposed solution provides guarantees for reliability, without sacrificing much economy.

II. System Model

We study a power grid with N_b buses in $\mathcal{B} = \{1, 2, ..., N_b\}$ interconnected by N_l transmission lines in $\mathcal{L} = \{1, 2, ..., N_l\}$. Each bus b could have some conventional generators, renewable supply, and demand.

Assume that the RAC stage is conducted every T time slots. The purpose of RAC is to ensure that future real-time dispatch decisions can always balance the supply and demand for all possible realizations of demand and renewable supply in the following T time slots.

Demand: We first model the demand side. In this work, we assume that the renewable supply will always be used to provide energy when available (and never curtailed). As a result, the renewable supply can be viewed as negative demand. Thus, we only need to focus on the net-demand, i.e., the total demand minus the total renewable supply, at each bus. Let $D_b(t)$ ($b \in \mathcal{B}, t = 1, 2, ..., T$) be the net-demand at bus b and time t. Then, the entire net-demand time-sequence can be denoted as $D(1{:}T) = \{D_b(1{:}T), b \in \mathcal{B}\}$, where $D_b(1{:}T) = \{D_b(t), t = 1, 2, ..., T\}$ is the net-demand time-sequence for each bus $b \in \mathcal{B}$.

Uncertainty: We now model the uncertainty in net-demand due to renewable supply, on which the multi-stage decisions must be computed. Like [7] and [9], we assume that the net-demand sequence $D(1{:}T)$ may be any sequence in a given uncertainty set $\mathcal{D}$. In practice, this uncertainty set is typically constructed from forecasts performed before time 1 [7]. However, at each time t, only the subsequence $D(1{:}t)$ is known, while the future subsequence $D(t+1{:}T)$ remains uncertain. While our proposed methodology can be applied to any form of uncertainty sets, for simplicity we will use the following form for the rest of the paper. The uncertainty set $\mathcal{D}$ consists of all $D(1{:}T)$ satisfying the following:

$$D_b^{\min}(t) \le D_b(t) \le D_b^{\max}(t), \tag{1}$$

$$\Delta_b^{\text{down}}(t_1, t_2) \le D_b(t_1) - D_b(t_2) \le \Delta_b^{\text{up}}(t_1, t_2), \tag{2}$$

where the parameters $D_b^{\min}(t)$ and $D_b^{\max}(t)$ denote the lower and upper limits, respectively, of the net-demand of bus b at time t, and the parameters $\Delta_b^{\text{up}}(t_1, t_2)$ and $\Delta_b^{\text{down}}(t_1, t_2)$ denote the maximum downward and upward rate of change for the net-demand of bus b. We note that, in a multi-stage setting, the constraint (2) can be used to refine the remaining uncertainty at time t. Specifically, we introduce the following notation: for $t \le t_1 \le t_2$, let

$$\begin{aligned}\mathcal{D}_{[t_1,t_2]|D(1:t)} \quad = \quad & \{D(t_1{:}t_2) | \text{there exists } D'(1{:}T) \in \mathcal{D}, \\ & \text{such that } D'(1{:}t) = D(1{:}t), D'(t_1{:}t_2) = D(t_1{:}t_2)\},\end{aligned} \tag{3}$$

which captures the remaining future uncertainty in the interval $[t_1, t_2]$, given the past net-demand trajectory $D(1{:}t)$. If $t = 0$, we will simplify the notation as $\mathcal{D}_{[t_1,t_2]}$, which is just the original uncertainty set $\mathcal{D}$ restricted to the time interval $[t_1, t_2]$. Note that $\mathcal{D}_{[t_1,t_2]|D(1:t)}$ is usually much smaller than $\mathcal{D}_{[t_1,t_2]}$.

Supply: We use $\mathcal{G} = \{1, 2, ..., N_g\}$ to denote the set of generators in the system, and use $P_g(t)$ to denote the power level at each generator g and each time slot t. Denote

$$\mathbf{P}(1{:}T) = \{\mathbf{P}_g(1{:}T), g \in \mathcal{G}\} = \{P_g(t), g \in \mathcal{G}, t = 1, 2, ..., T\}.$$

For a bus $b \in \mathcal{B}$, we use $\mathcal{G}_b \subseteq \mathcal{G}$ to denote the set of generators located at the bus b. Further, different generators have different power level and ramping constraints. Specifically, for each generator $g \in \mathcal{G}$, $P_g(t)$ must satisfy

$$P_g^{\min} \leq P_g(t) \leq P_g^{\max}, t = 1, 2, ..., T, \tag{4}$$

$$|P_g(t) - P_g(t-1)| \leq R_g, t = 2, 3, ..., T, \tag{5}$$

where R_g is the generator g's ramping capability in one time slot (typically 5 minutes [2]). Notice that when $R_g = P_g^{\max} - P_g^{\min}$, the generator can ramp to any level within its power range. In the rest of the paper, we refer this special case as "instantly fast" generators, or simply "fast" generators.

Other Constraints: Given the demand $D(1{:}T)$, the power dispatch decision $\mathbf{P}(1{:}T)$ must satisfy both the demand-supply balance constraints and the transmission limit constraints. The demand-supply balance constraints require that the total power generation must be equal to the total net-demand (here, we ignore the transmission loss), i.e.,

$$\sum_{g \in \mathcal{G}} P_g(t) = \sum_{b \in \mathcal{B}} D_b(t), t = 1, 2, ..., T. \tag{6}$$

To model the transmission limit constraints, we assume a simplified DC model [14]. Let $S = [S_{l,b}]$ be the shift factor of the power system, which is determined by the network topology and the reactance of the transmission lines. Then, the amount of power transmitting on line l at time slot t cannot exceed the power line l's transmission limit, TL_l, i.e.,

$$\left| \sum_{b=1}^{N_b} S_{l,b} \left(D_b(t) - \sum_{g \in \mathcal{G}_b} P_g(t) \right) \right| \leq \mathrm{TL}_l, \text{for any } t, l. \tag{7}$$

A. Real Time Dispatch and RAC

In practice, the dispatch decisions must respect *causality*, i.e., at each time t, the ISO can only make decisions based on the revealed net-demand subsequence $D(1{:}t)$ and the remaining uncertainty set $\mathcal{D}_{[t+1,T]|D(1:t)}$, but cannot depend on the exact values of $D(t+1{:}T)$. Due to the causality requirement, it is possible that, if an incorrect decision was made at an earlier time, then at a future time, no dispatch decisions can meet all the constraints. Thus, it is imperative that the decisions at each time take future uncertainty into account, so that such scenario will never occur. Towards this end, we introduce the following concept.

Definition 1. *Given an uncertainty set $\mathcal{D}$, we call $\pi(\mathcal{D})$[2] a causal real-time dispatch algorithm under $\mathcal{D}$, if at each time t, the dispatch decision $\mathbf{P}^{\pi(\mathcal{D})}(t) = \{P_g^{\pi(\mathcal{D})}(t), g \in \mathcal{G}\}$ produced by the algorithm $\pi(\mathcal{D})$ is a function of $D(1{:}t)$.*

Definition 2. *We say that a causal real-time dispatch algorithm $\pi(\mathcal{D})$ is* robust *for the uncertainty set $\mathcal{D}$, if and only if for all demand sequence $D(1{:}T) \in \mathcal{D}$ and all time $t = 1, ..., T$, the dispatch decision $\mathbf{P}^{\pi(\mathcal{D})}(t)$ produced by the algorithm $\pi(\mathcal{D})$ satisfies constraints (4)-(7).*

[2]Since $\mathcal{D}$ is a part of the problem formulation, the algorithm π may thus behave differently for different uncertainty sets $\mathcal{D}$.

The objectives of this work are then the following. First, at the RAC stage, given an uncertainty set $\mathcal{D}$, and a set of generators committed, we would like to know whether there exist causal real-time dispatch algorithms that are robust for the uncertainty set $\mathcal{D}$. Then, if the answer at the RAC stage is positive, we would like to find a causal real-time dispatch algorithm that is indeed robust for the uncertainty set $\mathcal{D}$. Note that unlike robust optimization [7], our formulation does not aim to minimize the worst-case future cost. As was discussed in the introduction and will be presented next, our solution produces "safe dispatch sets" that allow the operator to balance both reliability and economy.

B. Need to Handle Heterogeneity

Before we present our solution, we motivate our problem formulation through a simple example. Limitations of two-stage formulations have been reported in [9] (See another example in [15]). Here, we focus on the limitation of AARO in [9]. Specifically, [9] restricts the future decisions to be affine functions of the future input, i.e.,

$$P_g^{\mathrm{AARO}}(t) = w_g(t) + \sum_{b \in \mathcal{B}} W_{b,g}(t) D_b(t), \tag{8}$$

where $w_g(t)$ and $W_{b,g}(t)$ are coefficients computed in advance, subject to the constraints that the decisions in (8) must meet the constraints (4)-(7) for all realizations from the uncertainty set $\mathcal{D}$. In particular, $W_{b,g}(t)$ can be interpreted as the fraction of demand assigned to generator g from bus b. Unfortunately, the AARO approach can be inefficient in utilizing the complementary capabilities of heterogeneous generators, and hence its decisions can be overly conservative, as shown in the following example.

Example with Heterogeneous Generators: Let n be a positive parameter. We assume the following uncertainty set $\mathcal{D} = \{D(1:T) | 0 \leq D(t) \leq n^2 \text{ and } |D(t) - D(s)| \leq |t-s| + n, \forall t, s \leq T\}$. In other words, the net-demand $D(t)$ at any time can be between 0 and n^2. Further, $D(t)$ could jump by $n+1$ in one time-slot. However, if such jump indeed occurs, further changes of $D(t)$ along the same direction will have to be slower, i.e., at a long-term rate of 1 per time-slot. Now, consider a pair of generators: a slow generator with ramping rate $R = 1$ that can operate within the whole power range $[0, n^2]$, and a (instantly) fast generator that can operate at any power level in the smaller range $[0, 2n]$, but without ramping constraints. Intuitively, we can use the fast generator to meet the sudden jump, and use the slow generator to follow the slower long-term changes. In this way, a robust real-time dispatch algorithm does exist for this uncertainty set (see Remark 2 on how this conclusion can be verified). In contrast, if one applies AARO policy in (8) to each of the two generators, this uncertainty set can no longer be supported. To see this, note that no more than $W_{\text{slow}}(t) = \frac{1}{n+1}$ fraction of uncertainty can be assigned to the slow generator at any time. Otherwise, the ramping requirement in one time-slot will exceed $(n+1) \times \frac{1}{n+1} = 1$, which is the ramping limit of the slow generator. Similarly, no more than $W_{\text{fast}}(t) = \frac{2}{n}$ fraction of uncertainty can be assigned to the fast generator at any time.

Otherwise, the power level will exceed $n^2 \times \frac{2}{n} = 2n$, which is the power limit of the fast generator. Together, only about $\frac{3}{n}$ of the uncertainty can be supported by any affine policy.

In the following, we will present our solution for multi-stage decisions that not only correctly determines system reliability, but also avoids such inefficiency.

III. Maximally Robust Algorithm

In order to develop new policies that can more efficiently utilize a fleet of complementary generators, in this section we formulate what the "optimal" policy should be. Below, we first consider the real-time dispatch algorithm. Our goal is to formulate an "optimal" real-time dispatch algorithm that best utilizes any given set of generators, regardless of the RAC decision. The notion of optimality is defined below.

Definition 3. *Let $\Lambda = \{\mathcal{D}|$ There exists a causal real-time dispatch algorithm π_0 that is robust for the uncertainty set $\mathcal{D}\}$. A causal real-time dispatch algorithm π is said to be* maximally robust *if and only if $\pi(\mathcal{D})$ is robust for all the uncertainty sets $\mathcal{D} \in \Lambda$.*

In other words, if any other algorithm is robust for an uncertainty set $\mathcal{D}$, then a maximally robust algorithm π must also be robust for $\mathcal{D}$. Thus, a maximally robust algorithm is not dominated by any other algorithms in terms of reliability. It turns out that this problem of determining dynamic control decisions so that the system states remain in a desired set of trajectories has been studied by dynamic programming (see section 4.6.2 in [10]). Similar to the "target set" in [10, p197], we can introduce the notion of "safe dispatch sets" as follows:

Definition 4. *Given an uncertainty set $\mathcal{D}$, a demand history $D(1{:}t)$ and a power dispatch decision $\mathbf{P}(t)$, a causal real-time dispatch algorithm π is said to be* robust given $D(1{:}t)$ and $\mathbf{P}(t)$, *if and only if for any $D(t+1{:}T) \in \mathcal{D}_{[t+1,T]|D(1:t)}$, the algorithm π produces dispatch decisions $\{P_g^{\pi}(t_1), t_1 > t, g \in \mathcal{G}\}$ satisfying constraints (4)-(7) for all $t_1 > t$. (Note that (4)-(7) are defined for the entire range of t, but here $P_g^{\pi}(t_1)$ is only defined for $t_1 > t$.)*

Definition 5. *Given the demand history $D(1{:}t)$, a dispatch decision $\mathbf{P}(t)$ is safe if $\mathbf{P}(t)$ belongs to the safe dispatch set $\mathcal{F}(D(1{:}t))$, i.e.,*

$$\mathcal{F}(D(1{:}t)) = \{\mathbf{P}(t)|\mathbf{P}(t) \text{ can balance } D(t) \text{ subject to the constraints (4), (6) and (7), and there exists a causal algorithm } \pi \text{ that is robust given } D(1{:}t) \text{ and } \mathbf{P}(t)\}. \tag{9}$$

Intuitively, a maximally-robust algorithm simply needs to pick a dispatch decision at each time t from the safe dispatch set $\mathcal{F}(D(1{:}t))$. As in [10, p197], this safe dispatch set can be generated via backward induction. Then, the induction formula is given by $\mathcal{F}(D(1{:}T)) = A_T(D(T))$ and

$$\mathcal{F}(D(1{:}t)) = \left(\bigcap_{D(t+1)} f_t(\mathcal{F}(D(1{:}t+1)))\right) \bigcap A_t(D(t)), \tag{10}$$

where $A_t(D(t))$ is the set of generator decisions that can balance demand $D(t)$ and $f_t(A) = \{\mathbf{P}(t)|\ \exists \mathbf{P}(t+1) \in A,$ such that $|P_g(t+1) - P_g(t)| \le R_g\}$. The detailed proof of the induction formula (10) is available in technical report [16]. We now summarize the results that can be shown based on [10, p197]:

Proposition 6. *Given the uncertainty set $\mathcal{D}$, there exists a causal and robust real-time dispatch algorithm if and only if $\mathcal{F}(D(1)) \neq \emptyset$ for all $D(1) \in \mathcal{D}_1 \triangleq \{D(1)|D(1{:}T) \in \mathcal{D}\}$. Further, if $\mathcal{F}(D(1)) \neq \emptyset$ for all $D(1) \in \mathcal{D}_1$, then any algorithm in the following class is maximally robust:*

Step 1: *at time slot 1, pick an arbitrary dispatch decision $P(1) \in \mathcal{F}(D(1))$;*

Step 2: *at time slot $t > 1$, pick an arbitrary dispatch decision $\mathbf{P}(t) \in \mathcal{F}(D(1{:}t)) \bigcap C(\mathbf{P}(t-1))$, where*

$$C(\mathbf{P}(t-1)) = \{\mathbf{P}(t) : |P_g(t) - P_g(t-1)| \le R_g, g \in \mathcal{G}\} \tag{11}$$

is the set of dispatch decisions that can be reached at time t from the dispatch decision $\mathbf{P}(t-1)$ at time $t-1$.

The proof is available in [16]. According to Proposition 6, once we know how to calculate the safe dispatch set $\mathcal{F}(D(1{:}t))$, both the RAC decision (first part of Proposition 6) and the real-time dispatch decision (second part of Proposition 6) are solved. However, in general the complexity of the backward induction (10) is high because there exist uncountably many demand sequences. Thus, the backward induction is useful only for theoretical analysis.

IV. Computationally-Efficient Algorithms

In this paper, our goal is to design computationally-efficient algorithms that can approximate the optimal policy. Towards this end, we take a divide-and-conquer approach. First, recall from Section II-B that, for the case with one (instantly) fast generator and one slow generator, AARO may support a much lower level of uncertainty than the maximum possible. We thus focus on this case first. Surprisingly, we show in Section IV-A that in this special case, the safe-dispatch set can be precisely characterized in polynomial time, and thus the optimal dispatch decisions can be solved effectively. We then use this special case as *a key building block* for the general case in Section IV-B with multiple generators of arbitrary power level and ramping rates, and develop a general algorithm with polynomial time complexity. This algorithm can utilize complementary generators more efficiently than AARO, and thus support a higher level of renewable uncertainty with same amount of resources.

A. One Slow Generator + One Fast Generator

We first consider a one-bus two-generator case. We assume that the first generator is a slow generator, with time-varying generation limits $[P_{\text{slow}}^{\text{vmin}}(t), P_{\text{slow}}^{\text{vmax}}(t)]$, and time-varying up-ramping rate $R_{\text{slow}}^{\text{vup}}(t)$ and down-ramping rate $R_{\text{slow}}^{\text{vdown}}(t)$. Thus, the dispatched power level $P_{\text{slow}}^{\text{v}}(t)$ for this slow generator must satisfy

$$0 \le P_{\text{slow}}^{\text{vmin}}(t) \le P_{\text{slow}}^{\text{v}}(t) \le P_{\text{slow}}^{\text{max}}(t),$$

$$-R_{\text{slow}}^{\text{vdown}}(t) \le P_{\text{slow}}^{\text{v}}(t+1) - P_{\text{slow}}^{\text{v}}(t) \le R_{\text{slow}}^{\text{vup}}(t).$$

The second generator is a fast generator. We assume that this fast generator can generate both negative and positive power in the range $P^{\text{v}}_{\text{fast}}(t) \in [-r^{\text{v-}}_{\text{fast}}(t), r^{\text{v+}}_{\text{fast}}(t)]$, and it has no ramping constraint, i.e., the parameter R_g in (5) is $+\infty$.

Since there is only one bus, $D(t)$ becomes a scalar. Thus, if we know the dispatch level $P^{\text{v}}_{\text{slow}}(t)$ of the slow generator, we can immediately obtain the dispatch level $P^{\text{v}}_{\text{fast}}(t)$ of the fast generator through $P^{\text{v}}_{\text{fast}}(t) = D(t) - P^{\text{v}}_{\text{slow}}(t)$. Hence, in the following discussion, we will only focus on the dispatch level $P^{\text{v}}_{\text{slow}}(t)$ of the slow generator. We first assume that $\mathcal{F}(D(1{:}t)) \neq \emptyset$, and derive some necessary conditions that the generator parameters ($P^{\text{vmin}}_{\text{slow}}(t)$, $P^{\text{vmax}}_{\text{slow}}(t)$, $R^{\text{vdown}}_{\text{slow}}(t)$, $R^{\text{vup}}_{\text{slow}}(t)$, $r^{\text{v-}}_{\text{fast}}(t)$, $r^{\text{v+}}_{\text{fast}}(t)$) need to satisfy. We then show that these conditions are also sufficient. As a result, we will obtain a close-form formula for $\mathcal{F}(D(1{:}t))$.

1) Necessary Conditions for $\mathcal{F}(D(1{:}t)) \neq \emptyset$: The first set of necessary conditions are quite obvious and they simply check whether the lower/upper limits of the slow generator are consistent with its ramping speed.

Lemma 7. *(Parameter-checking condition) Given $D(1{:}t)$, if $\mathcal{F}(D(1{:}t)) \neq \emptyset$, then for any $t \leq t_0 \leq t_1$, the following constraints hold:*

$$P^{\text{vmin}}_{\text{slow}}(t_0) - \sum_{s=t_0}^{t_1-1} R^{\text{vdown}}_{\text{slow}}(s) \leq P^{\text{vmax}}_{\text{slow}}(t_1), \tag{12}$$

$$P^{\text{vmax}}_{\text{slow}}(t_0) + \sum_{s=t_0}^{t_1-1} R^{\text{vup}}_{\text{slow}}(s) \geq P^{\text{vmin}}_{\text{slow}}(t_1). \tag{13}$$

Clearly, if (12) is violated, then from any allowed power level at time t_0 (above $P^{\text{vmin}}_{\text{slow}}(t_0)$), the slow generator would have no way to ramp down to an allowed power level at t_1 (below $P^{\text{vmax}}_{\text{slow}}(t_1)$). Thus, $\mathcal{F}(D(1{:}t))$ would have been empty. The necessity of (13) is similar.

Even if conditions in (12) and (13) hold, the slow generator still may not be able to use all the power levels in $[P^{\text{vmin}}_{\text{slow}}(t_0), P^{\text{vmax}}_{\text{slow}}(t_0)]$. For instance, if $t_0 < t'$ and $P^{\text{vmin}}_{\text{slow}}(t_0) < P^{\text{vmin}}_{\text{slow}}(t') - \sum_{s=t_0}^{t'-1} R^{\text{vup}}_{\text{slow}}(s)$, then the slow generator should not use any power level below $P^{\text{vmin}}_{\text{slow}}(t') - \sum_{s=t_0}^{t'-1} R^{\text{vup}}_{\text{slow}}(s)$ at time t_0. Otherwise, it will not be able to ramp up to any allowed power level (above $P^{\text{vmin}}_{\text{slow}}(t')$) at time t'. Similarly, if $t' < t_0$, $P^{\text{vmin}}_{\text{slow}}(t_0)$ should not below $P^{\text{vmin}}_{\text{slow}}(t') - \sum_{s=t'}^{t_0-1} R^{\text{vdown}}_{\text{slow}}(s)$. Thus, we can define the "effective lower limit" and "effective upper limit" of the slow generator at time $t_0 \geq t$ as

$$\begin{aligned} P^{\text{eff-vmin}}_{\text{slow}}(t_0) \;&= \max\{\max_{t_0 < t' \leq T}\{P^{\text{vmin}}_{\text{slow}}(t') - \sum_{s=t_0}^{t'-1} R^{\text{vup}}_{\text{slow}}(s)\}, \\ &\max_{t \leq t' \leq t_0}\{P^{\text{vmin}}_{\text{slow}}(t') - \sum_{s=t'}^{t_0-1} R^{\text{vdown}}_{\text{slow}}(s)\}\}, \end{aligned} \tag{14}$$

$$\begin{aligned} P^{\text{eff-vmax}}_{\text{slow}}(t_0) \;&= \min\{\min_{t_0 < t' \leq T}\{P^{\text{vmax}}_{\text{slow}}(t') + \sum_{s=t_0}^{t'-1} R^{\text{vdown}}_{\text{slow}}(s)\}, \\ &\min_{t \leq t' \leq t_0}\{P^{\text{vmax}}_{\text{slow}}(t') + \sum_{s=t'}^{t_0-1} R^{\text{vup}}_{\text{slow}}(s)\}\}. \end{aligned} \tag{15}$$

Clearly, the power level of the slow generator at time t_0 should be within $[P^{\text{eff-vmin}}_{\text{slow}}(t_0), P^{\text{eff-vmax}}_{\text{slow}}(t_0)]$. The following necessary condition is then obvious.

Lemma 8. *(Capacity condition) Given $D(1{:}t)$, if $\mathcal{F}(D(1{:}t)) \neq \emptyset$, then for any $t_0 \geq t$, the following conditions must hold:*

$$\min_{D(t_0) \in \mathcal{D}_{t_0|D(1:t)}} \{D(t_0)\} \geq P^{\text{eff-vmin}}_{\text{slow}}(t_0) - r^{\text{v-}}_{\text{fast}}(t_0), \tag{16}$$

$$\max_{D(t_0) \in \mathcal{D}_{t_0|D(1:t)}} \{D(t_0)\} \leq P^{\text{eff-vmax}}_{\text{slow}}(t_0) + r^{\text{v+}}_{\text{fast}}(t_0). \tag{17}$$

In other words, no future demand can exceed the combined limits of the slow and fast generators.

While the above conditions are more obvious, the next condition is the key to capture the safety requirement in multi-stage decisions.

Lemma 9. *(Load-following condition) Given $D(1{:}t)$, if $\mathcal{F}(D(1{:}t)) \neq \emptyset$, then for any $t \leq t_0 \leq \min\{t_1, t_2\}$, the following condition must hold:*

$$\begin{aligned} &r^{\text{v+}}_{\text{fast}}(t_1) + \sum_{s=t_0}^{t_1-1} R^{\text{vup}}_{\text{slow}}(s) + r^{\text{v-}}_{\text{fast}}(t_2) + \sum_{s=t_0}^{t_2-1} R^{\text{vdown}}_{\text{slow}}(s) \\ &\geq \max_{D(1:t_0) \in \mathcal{D}_{[1,t_0]|D(1:t)}} \Big\{ \max_{D(t_1) \in \mathcal{D}_{t_1|D(1:t_0)}} \{D(t_1)\} - \\ &\qquad \min_{D(t_2) \in \mathcal{D}_{t_2|D(1:t_0)}} \{D(t_2)\} \Big\}. \end{aligned} \tag{18}$$

Proof. Given any net-demand sequence $D(1{:}t_0) \in \mathcal{D}_{[1,t_0]|D(1:t)}$, consider the dispatch decision ($P^{\text{v}}_{\text{slow}}(t_0)$, $P^{\text{v}}_{\text{fast}}(t_0)$) at time t_0. We need to ensure that for any time $t_1 \geq t_0$, the maximum demand is reachable. (If not, the safe dispatch set $\mathcal{F}(D(1{:}t))$ would have been empty.) Thus, we have

$$P^{\text{v}}_{\text{slow}}(t_0) + \sum_{s=t_0}^{t_1-1} R^{\text{vup}}_{\text{slow}}(s) + r^{\text{v+}}_{\text{fast}}(t_1) \geq \max_{D(t_1) \in \mathcal{D}_{t_1|D(1:t_0)}} \{D(t_1)\}. \tag{19}$$

Similarly, in order to reach the minimum demand at time $t_2 \geq t_0$, we must have

$$P^{\text{v}}_{\text{slow}}(t_0) - \sum_{s=t_0}^{t_2-1} R^{\text{vdown}}_{\text{slow}}(s) - r^{\text{v-}}_{\text{fast}}(t_2) \leq \min_{D(t_2) \in \mathcal{D}_{t_2|D(1:t_0)}} \{D(t_2)\}. \tag{20}$$

Then, we must have

$$\gamma^{\min}_{t_1}(D(1{:}t_0)) \leq P^{\text{v}}_{\text{slow}}(t_0) \leq \gamma^{\max}_{t_2}(D(1{:}t_0)), t_1, t_2 \geq t_0, \tag{21}$$

where $\gamma^{\min}_{t_1}(D(1{:}t_0)) \triangleq \max_{D(t_1) \in \mathcal{D}_{t_1|D(1:t_0)}}\{D(t_1)\} - \sum_{s=t_0}^{t_1-1} R^{\text{vup}}_{\text{slow}}(s) - r^{\text{v+}}_{\text{fast}}(t_1)$ and $\gamma^{\max}_{t_2}(D(1{:}t_0)) \triangleq \min_{D(t_2) \in \mathcal{D}_{t_2|D(1:t_0)}}\{D(t_2)\} + \sum_{s=t_0}^{t_2-1} R^{\text{vdown}}_{\text{slow}}(s) + r^{\text{v-}}_{\text{fast}}(t_2)$. Since (21) is true for all $D(1{:}t_0) \in \mathcal{D}_{[1,t_0]|D(1:t)}$, taking the worst case among all $D(1{:}t_0)$, we get (18). □

Remark 1. *Since the uncertainty set $\mathcal{D}$ only consists of linear constraints (1) and (2), it is easy to see that the terms "$\min_{D(t_0) \in \mathcal{D}_{t_0|D(1:t)}}\{D(t_0)\}$", "$\max_{D(t_0) \in \mathcal{D}_{t_0|D(1:t)}}\{D(t_0)\}$" in (16)-(17), and the right-hand-side of (18), are all convex optimization problems, and thus can be computed efficiently.*

2) Sufficiency of Conditions (12)-(13),(16)-(18): While the necessity of the above conditions (12)-(13),(16)-(18) are easy to follow, the next result is more surprising and it shows that these conditions are also sufficient for $\mathcal{F}(D(1{:}t)) \neq \emptyset$. Establishing this sufficiency is our first main contribution.

Theorem 10. *Given $D(1{:}t)$, if all the five conditions (12)-(13),(16)-(18) hold, then the safe dispatch set $\mathcal{F}(D(1{:}t))$ is not empty. Further, it can be explicitly expressed as follows:*

$$\begin{aligned}\mathcal{F}(D(1{:}t)) \quad = \quad & \{(P^{v}_{slow}(t), P^{v}_{fast}(t)) | P^{v}_{slow}(t) \in h(D(1{:}t)),\\ & P^{v}_{slow}(t) + P^{v}_{fast}(t) = D(t)\}. \end{aligned} \tag{22}$$

The interval $h(D(1{:}t))$ above can be computed as

$$\begin{aligned} h(D(1{:}t)) = \Bigg[& \max\left\{ P^{\textit{eff-vmin}}_{slow}(t), \max_{t_1=t}^{T} \gamma^{min}_{t_1}(D(1{:}t)) \right\},\\ & \min\left\{ P^{\textit{eff-vmax}}_{slow}(t), \min_{t_1=t}^{T} \gamma^{max}_{t_1}(D(1{:}t)) \right\} \Bigg], \end{aligned} \tag{23}$$

where $\gamma^{min}_{t_1}(D(1{:}t))$ and $\gamma^{max}_{t_1}(D(1{:}t))$ are defined below (21).

Note that the upper and lower limits in $h(D(1{:}t))$ are simply combinations of the limits in (21) and the effective limits $[P^{\text{eff-vmin}}_{\text{slow}}(t), P^{\text{eff-vmax}}_{\text{slow}}(t)]$. Thus, it naturally produces an outer bound on $\mathcal{F}(D(1{:}t))$. To show that $h(D(1{:}t))$ produces the exact form of $\mathcal{F}(D(1{:}t))$ as in (22), we will have to show that, for any $P(t) = (P^{\text{v}}_{\text{slow}}(t), P^{\text{v}}_{\text{fast}}(t))$ within the right hand side of (22), we can construct a causal real-time dispatch algorithm π (like in Prop. 6), such that this algorithm π is robust given $D(1{:}t)$ and $P(t)$. The detailed construction is in the proof of Theorem 10 (see Appendix A).

Remark 2. *In Theorem 10, we have derived the precise characterization of the safe dispatch set for a single-bus system with a pair of slow and fast generators. To see how our proposed approach can achieve a larger safe dispatch set than AARO, we use the Example in Section II-B. One can verify that such a configuration satisfies the conditions (12)-(13) and (16)-(18) in Theorem 10. (See details in our technical report [16]). Thus, the entire uncertainty set can be handled by this pair of generators. In contrast, as we have illustrated in Section II-B, if any affine policy is used, only about $\frac{3}{n}$ of the uncertainty can be supported. As n increases, the gap between the affine policies and our precise characterization grows as $O(n)$. We refer the readers to additional simulations in Section V to further illustrate the difference.*

B. Multiple Buses + Multiple Generators

In this section, we study the general case with multiple generators of arbitrary power levels and ramping rates, which unfortunately does not admit exact closed-form characterizations for the safe dispatch set. Instead, we aim to utilize a suitable subset of the true safe dispatch set.

Our basic idea is *demand splitting*, i.e., fractions of the future net-demand uncertainty are sent to the generators according to pre-computed *splitting factors*. However, assigning a splitting factor for each physical generator will lead to severely-reduced safe dispatch sets, because it fails to leverage the complementary capabilities of generators, an example of which is shown in Section II-B. This observation thus motivates us to consider the following idea of *virtual demand splitting* (VDS). Based on the actual physical generators, we will conceptually form pairs of virtual fast generator (VFG) and virtual slow generator (VSG). A fraction (i.e., the splitting factor) of net-demand uncertainty is then sent to such a virtual generator pair (VGP). For each VGP, we can then use the two-generator characterization in Section IV-A. By carefully forming the VGPs and choosing the splitting factors, this procedure will result into a subset $\mathcal{F}^{\text{VDS}}(D(1{:}t))$ of the true safe dispatch set $\mathcal{F}(D(1{:}t))$, which can then be used for RAC and real-time dispatch to ensure robust operations.

Since our ultimate goal is real-time dispatch and RAC, below we first directly give the formulation (24) of the real-time dispatch decision following the above procedure, which produces economic dispatch $\mathbf{P}(t)$ at time t that is also safe for future uncertainty at all $t_0 > t$, given history $D(1{:}t$. The formulations of RAC and $\mathcal{F}^{\text{VDS}}(D(1{:}t))$ are similar and will be given next.

$$\underset{Z(t)}{\text{minimize}} \quad Cost(\mathbf{P}(t)) \tag{24a}$$

subject to

$$R^{\text{up}}_{\text{slow,g}}(t_0) + R^{\text{v+}}_{\text{fast},g}(t_0) = R_g, \quad \forall t_0 \geq t, \forall g \in \mathcal{G}, \tag{24b}$$

$$R^{\text{down}}_{\text{slow,g}}(t_0) + R^{\text{v-}}_{\text{fast},g}(t_0) = R_g, \quad \forall t_0 \geq t, \forall g \in \mathcal{G}, \tag{24c}$$

$$P^{\text{vmax}}_{\text{slow,g}}(t_0) + R^{\text{v+}}_{\text{fast},g}(t_0) = P^{\text{max}}_{\text{g}}, \quad \forall t_0 \geq t, \forall g \in \mathcal{G}, \tag{24d}$$

$$P^{\text{vmin}}_{\text{slow,g}}(t_0) - R^{\text{v-}}_{\text{fast},g}(t_0) = P^{\text{min}}_{\text{g}}, \quad \forall t_0 \geq t, \forall g \in \mathcal{G}, \tag{24e}$$

$$\sum_{g \in \mathcal{G}_b} R^{\text{v+}}_{\text{fast},g}(t_0) \geq \sum_{g \in \mathcal{G}_b} r^{\text{v+}}_{\text{fast},g}(t_0), \quad \forall t_0 \geq t, \forall b \in \mathcal{B}, \tag{24f}$$

$$\sum_{g \in \mathcal{G}_b} R^{\text{v-}}_{\text{fast},g}(t_0) \geq \sum_{g \in \mathcal{G}_b} r^{\text{v-}}_{\text{fast},g}(t_0), \quad \forall t_0 \geq t, \forall b \in \mathcal{B}, \tag{24g}$$

$$\sum_{b \in \mathcal{B}} D^{\text{main}}_{b}(t_0) = \sum_{g \in \mathcal{G}} P^{\text{main}}_{\text{VGP},g}(t_0), \quad \forall t_0 \geq t, \tag{24h}$$

$$\sum_{g \in \mathcal{G}} \eta_{b,g} = 1, \qquad \forall b \in \mathcal{B}, \tag{24i}$$

$$\begin{aligned} & \Big| \sum_{b \in \mathcal{B}} S_{l,b} \Big(D_b(t_0) - \sum_{g \in \mathcal{G}_b} D_g(t_0) \Big) \Big| \leq \text{TL}_l, \forall l \in \mathcal{L}, \\ & \quad \text{for all } D_b(t_0) \in \mathcal{D}_{t_0|D(1:t)},\ D_g(t_0) \text{ in (25)}, \forall t_0 \geq t \end{aligned} \tag{24j}$$

$$\begin{aligned} & \{R^{\text{up}}_{\text{slow,g}}(t), R^{\text{down}}_{\text{slow,g}}(t), r^{\text{v+}}_{\text{fast,g}}(t), r^{\text{v+}}_{\text{fast,g}}(t), P^{\text{main}}_{\text{VGP,g}}(t), \vec{\eta}\} \\ & \quad \text{satisfy constraints (12)-(18)}, \quad \forall \text{ VGP pair } g \end{aligned} \tag{24k}$$

$$\big(P^{\text{v}}_{\text{slow,g}}(t), P^{\text{v}}_{\text{fast,g}}(t)\big) \in \mathcal{F}^{Z(t)}_{g}(D(1:t)), \quad \forall g \in \mathcal{G}, \tag{24l}$$

$$-R^{\text{v-}}_{\text{fast,g}}(t) \leq P_g(t) - P^{\text{v}}_{\text{slow,g}}(t) \leq R^{\text{v+}}_{\text{fast,g}}(t), \ \forall g \in \mathcal{G}, \tag{24m}$$

$$\sum_{g \in \mathcal{G}_b} P_g(t) = \sum_{g \in \mathcal{G}_b} (P^{\text{v}}_{\text{slow,g}}(t) + P^{\text{v}}_{\text{fast,g}}(t)), \quad \forall b \in \mathcal{B}, \tag{24n}$$

$$|P_g(t) - P_g(t-1)| \leq R_g, \qquad \text{only if } t \geq 2, \forall g \in \mathcal{G}. \tag{24o}$$

The set of problem variables is $Z(t) = \{R^{\text{up}}_{\text{slow,g}}(t_0), R^{\text{down}}_{\text{slow,g}}(t_0), R^{\text{v+}}_{\text{fast,g}}(t_0), R^{\text{v-}}_{\text{fast,g}}(t_0), r^{\text{v+}}_{\text{fast,g}}(t_0), r^{\text{v+}}_{\text{fast,g}}(t_0), P^{\text{main}}_{\text{VGP,g}}(t_0), \eta_{b,g}, P_g(t), P^{v}_{\text{slow},g}(t), P^{v}_{\text{fast},g}(t), \forall g \in \mathcal{G}, \forall b \in \mathcal{B}, \forall t_0 \geq t\}$, which are explained below. In (24a), the objective is to minimize the generation cost at current time t, where $Cost(\cdot)$ is a convex energy-cost function. (24b)-(24e) are the constraints that define virtual

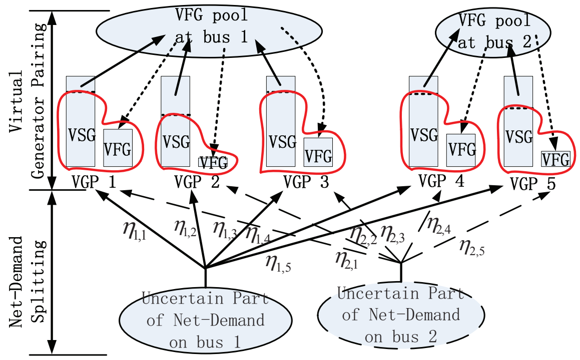


Fig. 1. Illustration of the VDS approach

generators, i.e., each physical generator $g \in \mathcal{G}_b$ contributes $R^{\text{v+}}_{\text{fast},g}$ and $R^{\text{v-}}_{\text{fast},g}$ from its ramp limit R_g to a VFG pool $\{\sum_{g\in\mathcal{G}_b} R^{\text{v+}}_{\text{fast},g}, -\sum_{g\in\mathcal{G}_b} R^{\text{v-}}_{\text{fast},g}\}$ on its corresponding bus b. The remaining power output range $(P^{\text{vmax}}_{\text{slow,g}}, P^{\text{vmin}}_{\text{slow,g}})$ and ramping rates $(R^{\text{up}}_{\text{slow},g}, R^{\text{down}}_{\text{slow},g})$ together form a VSG. Then, in (24f)-(24g), each VSG is paired with a VFG $(r^{\text{v+}}_{\text{fast},g}, r^{\text{v-}}_{\text{fast},g})$ from the VFG pool, to form a VGP (see Fig. 1). (24h) is the power demand-supply balance condition for the main part of the net-demand, which is given by $D^{\text{main}}_b(t_0) = (\max\{D_b(t_0)\} + \min\{D_b(t_0)\})/2$. For the remaining uncertain demand $(D_b(t_0) - D^{\text{main}}_b(t_0))$, we use affine splitting according to the splitting factors $\eta_{b,g}$ in (24i), such that (similar to (8), see Fig. 1)

$$D_g(t_0) = P^{\text{main}}_{\text{VGP,g}}(t_0) + \sum_{b\in\mathcal{B}} \eta_{b,g}(D_b(t_0) - D^{\text{main}}_b(t_0)), \quad \forall t_0 \geq t, \forall g \in \mathcal{G}. \quad (25)$$

$D_g(t_0)$ is then the total demand that VGP g must meet. (24j)-(24k) are the constraints that this splitting of uncertain demand must satisfy. (24j) ensures that the transmission-line limit is met for each possible value of $D_b(t_0) \in \mathcal{D}_{t_0|D(1:t)}$ and $D_g(t_0)$ according to (25), $\forall t_0 \geq t$. (24k) ensures that the exact safe dispatch set for each VGP g (which we characterized in Theorem 10 in Section IV-A) is non-empty according to the limits of VGP g and the uncertainty level of $D_g(t)$ from (25). This ensures that, for each VGP g, the lower limit of $h(\cdot)$ in (23) is no greater than the correspoding upper limit. Finally, (24l)-(24o) choose a safe dispatch decision $\mathbf{P}(t)$ for time t. In particular, (24l) chooses the power output levels of each VSG/VFG pair, $P^{\text{v}}_{\text{slow,g}}(t)$ and $P^{\text{v}}_{\text{fast,g}}(t)$, from the safe dispatch set $\mathcal{F}^{Z(t)}_g(D(1{:}t))$ of the VGP, which is given in Theorem 10. Then, (24m) and (24n) map from $(P^{\text{v}}_{\text{slow,g}}(t), P^{\text{v}}_{\text{fast,g}}(t)) \in \mathcal{F}^{Z(t)}_g(D(1{:}t))$ for all VGPs g, back to the dispatch decisions $\mathbf{P}(t)$ on real generators. Last but not the least, (24o) ensures that the safe dispatch decision $\mathbf{P}(t)$ is within the ramp limit from previous dispatch decision $\mathbf{P}(t{-}1)$.

Remark 3. *Note that* (24j)-(24l) *can be written as a polynomial number of convex constraints (see technical report [16]).*

While (24) focuses on real-time dispatch, similar formulations can be developed for characterizing the safe dispatch subset $\mathcal{F}^{\text{VDS}}(D(1{:}t))$ itself or for RAC. Specifically, the subset $\mathcal{F}^{\text{VDS}}(D(1{:}t))$ can be written as

$$\mathcal{F}^{\text{VDS}}(D(1{:}t)) = \{\mathbf{P}(t) | \text{There exists } Z(t) \text{ such that (24b)-(24n) holds}\}. \quad (26)$$

For RAC, we can simply use constraints (24b)-(24k) based on the original uncertainty set $\mathcal{D}$ and for all $t_0 \geq 1$. Further, we can minimize any trivial objective function subject to the constraints (24b)-(24k). If the minimization problem produces a finite value, we can conclude that the safe dispatch subset is non-empty, and the system must be safe for RAC.

Remark 4. *We note that our idea of demand splitting shares some similarity to the choice of AARO in [9]. However, there are two key differences. First, we assign a splitting factor for each pair of VSG and VFG (rather than for each generator). As we argue in Section IV-A (see Remark 2), the additional flexibility by using VGPs will likely enlarge the obtained subset* $\mathcal{F}^{VDS}(D(1{:}t))$. *Second, at each time, we reformulate the constraints* (24b)-(24o) *based on current and future uncertainty. In other words, our real-time dispatch decision is not restricted by the day-ahead parameters. In contrast, the real-time dispatch decisions in [9] are affected by the day-ahead decisions. Specifically, in the "policy-guided robust ED model" in [9], the real-time dispatch decision at each time slot is constrained in such a way that, for all possible renewable realization in the next time slot, the current dispatch level for each generator must be able to ramp to its power level for the next time-slot computed by the day-ahead affine policy. As a result, the real-time decisions of [9] is often more constrained and less economical. As we will show in the simulation results in Section V, this difference will lead to a reduction in energy costs for our proposed policy.*

V. Computational Results

In this section, we present the MATLAB simulation results to demonstrate the effectiveness of our proposed VDS approach on two different settings: a single-bus two-generator case and a more realistic case with a richer pool of generators.

A. A Single-bus Two-generator Case

We first demonstrate that, for a a simple single-bus two-generator setting without transmission constraints (similar to Section IV-A), our proposed algorithm can produce a larger safe dispatch set compared to AARO based approaches. The parameters of generators are summarized in the following Table I. Note that generator A represents the "slow" generator, and generator B represents the "fast" generator.

TABLE I
LISTS OF GENERATORS

Type	Generator Limit	Ramping Rate
A	0-500MW	12MW/slot
B	0-30MW	30MW/slot

We use the demand data from Elia [17], Belgium's electricity transmission system operator. Specifically, we scale down the load data from 6am to 9am on 01/01/2015 by 15 times

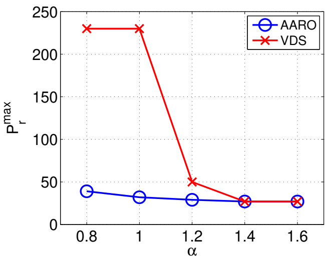


Fig. 2. The maximum wind power $P_r^{\max}$ that can be supported at different levels of wind variability α using the AARO approach in [9] and our proposed VDS algorithm.

and feed it into our single-bus power system (see Fig. 3(a)). For the wind supply, we create a parameterized uncertainty set so that we can easily experiment with different levels of wind output and variability. Specifically, the renewable power P_r varies within $[0, P_r^{\max}]$, and the maximum variation of renewable energy is given by $\Delta_w^{\text{up}}(\Delta t) = \alpha[20 + 5(\Delta t - 1)]$ and $\Delta_w^{\text{down}}(\Delta t) = \alpha[-20 - 5(\Delta t - 1)]$, where the positive parameter α controls the variability of the wind-output.

We compare our proposed approach at the RAC stage with state-of-the-art AARO approach [9] under different values of $P_r^{\max}$ and α. In Fig. 2, for each value of α, we plot the maximum value of $P_r^{\max}$ at which each algorithm determines the system as reliable ($P_r^{\max}$ can be viewed as the size of safe dispatch set). As shown in Fig. 2, our proposed VDS algorithm always performs no worse than AARO, and can ensure reliability at significantly higher levels of wind than AARO when α is small. The performance gain is over 5 times when the wind variation is small (e.g., $\alpha = 0.8$ and 1). Note that this observation is consistent with our earlier analysis in Remark 2 of Section IV-A. The above results thus confirm our discussion earlier that our proposed approach can yield a larger safe dispatch set compared to AARO. In the next section, we will show that such enhancement in reliability will transform into economical gains even in the case where AARO determines the system as reliable.

B. IEEE 30-bus Test Case

Next, we evaluate the VDS algorithm on a standard IEEE 30-bus system [18]. The parameters of different generator types and location information are listed in Table II. Here, Type I, II and III of generators can be viewed as "slow" generators with different capabilities, while Type IV generators can be viewed as "fast" generators. By pairing "slow" and "fast" generators on Bus 2, 13, 22 and 23, we expect that our proposed algorithm can outperform other existing approaches in the literature. We take the load data from Section V-A and split it evenly into two parts, which are then fed into buses 2 and 3. The wind data is also from 6am to 9am on 01/01/2015. We apply a linear transformation to scale down the data (see Fig. 3(b), the curve labeled "Real Renewable"), and feed it to bus 3. Note that the load data is more predictable than the wind data. Hence, for simplicity, we assume that the exact values of the load are known at the RAC stage, and thus the uncertainty all comes from the wind energy. The uncertainty set of the wind is modeled according to (1) and (2), and its parameters are estimated from historical data. Specifically, the upper bound and the lower bound are shown in Fig. 3(b), and the maximum variation $\Delta_w^{\text{up}}(\Delta t) = \Delta_w^{\text{up}}(t, t + \Delta t)$ and $\Delta_w^{\text{down}}(\Delta t) = \Delta_w^{\text{down}}(t, t + \Delta t)$ of wind (here we assume that the maximum variation only depends on the time difference) are shown in Fig.3(c).

TABLE II
LISTS OF FOSSIL-FUELED GENERATORS

Type	Generator Limits	Ramping Rate	Price	Bus Location
I	60-140MW	0.2MW/min	48$/MWh	1
II	21-60MW	0.2MW/min	56$/MWh	2, 23
III	50-160MW	0.2MW/min	60$/MWh	13, 22
IV	0-8MW	0.5MW/min	80$/MWh	2,13,22,23,27

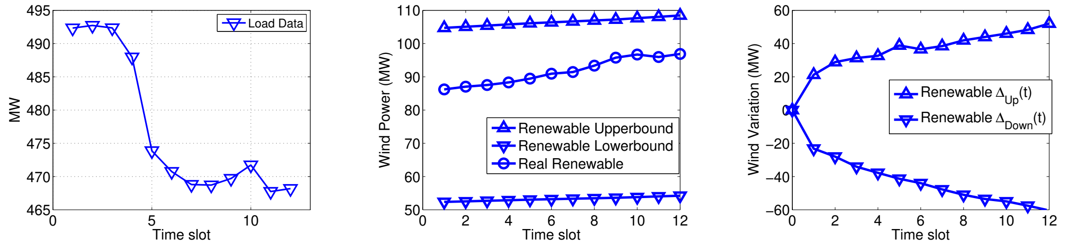
We first compare the decisions in the RAC stage. We compare VDS with AARO and the two-stage formulation. To compare these three approaches, we scale up the wind energy (both its bounds and maximum variation) by a scaling factor ranging from 0.4 to 1.4. In the middle three rows of Table III, for each renewable scale, we report whether the three approaches for RAC (i.e., VDS, AARO and two-stage formulation) will determine the system as reliable day-ahead. We make the following observations. First, note that AARO-RAC and VDS-RAC determine the system as reliable up to a renewable scale of 1.0 and 1.3, respectively. Thus, under the VDS algorithm, the system can support significantly more renewable than under AARO, even though the set of resources is the same. Second, the two-stage formulation (2-Stage RAC) declares the system as reliable at an even higher renewable scale of 1.4. However, as we discussed earlier, such assurance could be misleading because the real system is not two-staged. Indeed, the two-stage formulation by itself does not provide a way to dispatch generators in real time. As we will show shortly, none of the real-time economic dispatch algorithms that we try can meet all grid constraints in real time under such a high renewable scale of 1.4.

Specifically, in the last three rows of Table III, we report the total real-time generation costs (i.e., the summation of the cost at all times) under VDS-ED, AARO-ED (or "policy-guided ED" in [9]) and the standard ED (which minimizes the energy cost in the current slot). A value of "Inf" in these rows corresponds to the case when no feasible dispatch decisions meeting all grid constraints (4)-(7) can be found. We observe that, the system would be unreliable under the

TABLE III
DAY-AHEAD RAC AND REAL TIME DISPATCH COST

Wind Scale	0.4	0.6	0.8	1.0	1.3	1.4
VDS-RAC	yes	yes	yes	yes	yes	no
AARO-RAC	yes	yes	yes	yes	no	no
2-Stage RAC	yes	yes	yes	yes	yes	yes
VDS-ED	$72,833	$69,624	$66,467	$63,316	$58,955	Inf
AARO-ED	$73,076	$70,002	$66,819	$63,668	Inf	Inf
Standard-ED	$72,800	$69,542	Inf	Inf	Inf	Inf

Note: In row 2 to 4, "yes" or"no" means whether the system is determined as reliable day-ahead by the corresponding RAC policy.



(a) The system load data are obtained from Elia, from 6am to 9am on 01/01/2015.

(b) Renewable energy and its upper/lower bounds (from Elia wind from 6am to 9am on 01/01/2015).

(c) The maximum variation of renewable energy $\Delta_w^{\text{up}}(\Delta t)$ and $\Delta_w^{\text{down}}(\Delta t)$ (based on trace in Dec. 2014).

Fig. 3. Simulation data: (a) load; (b) wind and its range; (c) maximum variation of wind.

standard ED algorithm in real time once the scaling factor is beyond 0.6. In contrast, for VDS, whenever VDS-RAC reports that the system is reliable day-ahead, which is the case up to a renewable scale of 1.3, VDS-ED is guaranteed to maintain reliability in real-time. Similarly, AARO-ED can maitain real-time reliability whenever AARO-RAC declares the system as reliable day-ahead, albeit at a lower renewable scale up to 1.0. At renewable scale above 1.3, none of the three ED algorithms can maintain real-time reliability. We caution that, at this level of renewable uncertainty, we do not know whether there exist causal real-time dispatch algorithms that can maintain real-time reliability (because the VDS-ED algorithm is based on a subset of the true safe dispatch set). Nonetheless, this example illustrates that the two-stage formulation alone can easily lead to incorrect conclusions on system reliability.

Finally, we compare the numerical values in the last three rows in Table III. We observe that, when the standard ED algorithm can meet all grid constraints (scaling factor = 0.4 and 0.6), our VDS-ED achieves similar costs. This confirms our earlier discussion that VDS-ED only intervenes the ED decisions when necessary. On the other hand, when the standard ED algorithm fails to provide safe dispatch decisions (for scaling factor from 0.6 to 1.3), our proposed VDS-ED can still guarantee reliable grid operations, which demonstrates the importance of checking the safe dispatch set. Then, we compare VDS-ED with AARO-ED. Even though AARO [9] can achieve grid reliability up to a scaling factor of 1.0, its total fuel costs are always higher than those under VDS-ED. We believe that there are two reasons for the inefficiency of AARO-ED. First, as we discussed earlier in Remark 2 in Section IV-A, VDS in general yields a larger safe dispatch set than AARO. Hence, it can choose more-economic dispatch decisions. Second, as we discussed in Remark 4 of Section IV-B, the policy-guided ED [9] based on AARO imposes further restrictions on the ED decisions based on day-ahead decisions. Both factors lead to higher costs for AARO. Finally, at scaling factor of 1.3, VDS-ED is able to guarantee reliability, while AARO-ED fails. In summary, we conclude that VDS-ED achieves a higher level of reliability than the AARO-based ED algorithm and the standard ED algorithm, without sacrificing much economy.

C. Computational Complexity

To provide some insights on the computational complexity, we solve the same problem instance using our proposed VDS algorithm and AARO, for a standard IEEE 30-bus test system and an IEEE 118-bus test system [18]. We run the simulations using MATLAB/CVX with Mosek solver on a workstation with Intel(R) Core(TM) i7-6700 CPU @ 3.40GHz, 16GB Memory, and Linux CentOS. We simulate for a horizon of $T = 12$ time slots and the running times are summarized in Table IV. The results show that the computation time of our proposed VDS algorithm is about 4 times that of AARO for both instances, and the computation time only doubles when the system size increases from 30 to 118 buses. Hence, this suggests that our proposed solution can potentially scale to large systems, thanks to the fact that the proposed formulation of VDS-RAC and VDS-ED are convex.

TABLE IV
THE RUNNING TIME COMPARISON OF VDS AND AARO

	30-bus	118-bus
AARO-RAC	0.91 sec	1.72 sec
VDS-RAC	3.92 sec	8.40 sec

VI. CONCLUSION

We study online multi-stage decisions to ensure reliable grid operations under high renewable uncertainty. Using the concept of safe dispatch sets, we develop a computationally efficient algorithm that can provably guarantee grid reliability. Our proposed solution achieves a larger safe dispatch set compared to state-of-the-art multi-stage grid operation based on AARO policy [9]. Our simulation results show that our VDS algorithm outperforms AARO in both RAC and real-time dispatch.

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Appendix A
Proof of Theorem 10

We use I to denote the set on the right-hand-side of (22). The key in the proof is to design a causal real-time dispatch algorithm π that is robust given $D(1{:}t)$, starting from any $\mathbf{P}(t) = (P^{\text{v}}_{\text{slow}}(t), P^{\text{v}}_{\text{fast}}(t))$ within the set I. (Note that any $\mathbf{P}(t) \in I$ can balance the demand $D(t)$ at time t.) The detailed causal real-time dispatch algorithm is constructed as follows:

Algorithm π: At any time slot $t_0 > t$, pick an arbitrary dispatch decision $P^{\text{v}}_{\text{slow}}(t_0) \in h(D(1{:}t_0)) \bigcap C(P^{\text{v}}_{\text{slow}}(t_0-1))$, and set $P^{\text{v}}_{\text{fast}}(t_0) = D(t_0) - P^{\text{v}}_{\text{slow}}(t_0)$. Here, $h(D(1{:}t_0))$ is defined similarly to $h(D(1{:}t))$ as:

$$h(D(1{:}t_0)) = \left[\max\left\{P^{\text{eff-vmin}}_{\text{slow}}(t_0), \max_{t_1=t_0}^{T} \gamma^{\min}_{t_1}(D(1{:}t_0))\right\},\right. \left.\min\left\{P^{\text{eff-vmax}}_{\text{slow}}(t_0), \min_{t_1=t_0}^{T} \gamma^{\max}_{t_1}(D(1{:}t_0))\right\}\right]; \quad (27)$$

$C(P^{\text{v}}_{\text{slow}}(t_0-1))$ is the set of slow-generator output levels that can be reached at time t_0 from its output $P^{\text{v}}_{\text{slow}}(t_0-1)$ at time t_0-1, which is defined similar to "$C(\mathbf{P}(t-1))$" in (11) as:

$$C(P^{\text{v}}_{\text{slow}}(t_0-1)) = \{P^{\text{v}}_{\text{slow}}(t_0) : -R^{\text{vdown}}_{\text{slow}}(t_0-1) \le P^{\text{v}}_{\text{slow}}(t_0) - P^{\text{v}}_{\text{slow}}(t_0-1) \le R^{\text{vup}}_{\text{slow}}(t_0-1)\}. \quad (28)$$

In order to show that the above algorithm π is robust given $D(1{:}t)$ and $\mathbf{P}(t)$, we only need to show that we can always find such $P^{\text{v}}_{\text{slow}}(t_0)$ and $P^{\text{v}}_{\text{fast}}(t_0)$ within their respective limits. Thus, it suffices to prove the following three claims: As long as the conditions (12)-(13),(16)-(18) hold, we must have

1) $h(D(1{:}t_0)) \neq \emptyset$ for all $t_0 \ge t$.
2) $P^{\text{v}}_{\text{fast}}(t_0) = D(t_0) - P^{\text{v}}_{\text{slow}}(t_0) \in [-r^{\text{v-}}_{\text{fast}}(t_0), r^{\text{v+}}_{\text{fast}}(t_0)]$, for all $P^{\text{v}}_{\text{fast}}(t_0) \in h(D(1:t_0))$.
3) $h(D(1{:}t_0)) \bigcap C(P^{\text{v}}_{\text{slow}}(t_0-1)) \neq \emptyset$ for all $t_0 > t$.

Further, once we can show that Claim 1 and Claim 2 hold for $t_0 \ge t$, if we let $t_0 = t$, we immediately have $I \neq \emptyset$. Theorem 10 then follows. In the following three subsections, we will prove the above three claims.

A. Claim 1

$h(D(1{:}t_0)) \neq \emptyset$ is equivalent to the four inequalities below:

1) $P^{\text{eff-vmin}}_{\text{slow}}(t_0) \le P^{\text{eff-vmax}}_{\text{slow}}(t_0)$; (29)
2) $P^{\text{eff-vmin}}_{\text{slow}}(t_0) \le \gamma^{\max}_{t_2}(D(1{:}t_0))$, for all $t_2 \ge t_0$; (30)
3) $\gamma^{\min}_{t_1}(D(1{:}t_0))\} \le P^{\text{eff-vmax}}_{\text{slow}}(t_0)$, for all $t_1 \ge t_0$; (31)
4) $\gamma^{\min}_{t_1}(D(1{:}t_0))\} \le \gamma^{\max}_{t_2}(D(1{:}t_0))$, for all $t_1, t_2 \ge t_0$, (32)

which follow from (12), (13), and (16)- (18), respectively. The detailed proof is straightforward but lengthy, and is available in our technical report [16].

B. Claim 2

Since $P^{\text{v}}_{\text{slow}}(t_0) \in h(D(1{:}t_0))$, we must have that

$$\gamma^{\min}_{t_0}(D(1{:}t_0)) \le P^{\text{v}}_{\text{slow}}(t_0) \le \gamma^{\max}_{t_0}(D(1{:}t_0)).$$

Note that $\gamma^{\min}_{t_0}(D(1{:}t_0)) = D(t_0) - r^{\text{v+}}_{\text{fast}}(t_0)$ and $\gamma^{\max}_{t_0}(D(1{:}t_0)) = D(t_0) + r^{\text{v-}}_{\text{fast}}(t_0)$, we then have $P^{\text{v}}_{\text{fast}}(t_0) = D(t_0) - P^{\text{v}}_{\text{slow}}(t_0) \in [-r^{\text{v-}}_{\text{fast}}(t_0), r^{\text{v+}}_{\text{fast}}(t_0)]$.

C. Claim 3

In order to show that $h(D(1{:}t_0)) \bigcap C(P^{\text{v}}_{\text{slow}}(t_0-1)) \neq \emptyset$, it suffices to show that the following two inequalities

$$\max\left\{P^{\text{eff-vmin}}_{\text{slow}}(t_0), \max_{t_1=t_0}^{T} \gamma^{\min}_{t_1}(D(1{:}t_0))\right\} \le P^{\text{v}}_{\text{slow}}(t_0-1) + R^{\text{vup}}_{\text{slow}}(t_0-1), \text{ and} \quad (33)$$

$$\min\left\{P^{\text{eff-vmax}}_{\text{slow}}(t_0), \min_{t_1=t_0}^{T} \gamma^{\max}_{t_1}(D(1{:}t_0))\right\} \ge P^{\text{v}}_{\text{slow}}(t_0-1) - R^{\text{vdown}}_{\text{slow}}(t_0-1). \quad (34)$$

hold for any $P^{\text{v}}_{\text{slow}}(t_0-1) \in h(D(1{:}t_0-1))$. In the following, we only prove (33), while (34) can be proved similarly. To prove (33), it suffices to prove:

$$\max\left\{P^{\text{eff-vmin}}_{\text{slow}}(t_0), \max_{t_1=t_0}^{T} \gamma^{\min}_{t_1}(D(1{:}t_0))\right\} - R^{\text{vup}}_{\text{slow}}(t_0-1) \le \max\left\{P^{\text{eff-vmin}}_{\text{slow}}(t_0-1), \max_{t_1=t_0-1}^{T} \gamma^{\min}_{t_1}(D(1{:}t_0-1))\right\} \quad (35)$$

To prove (35), it suffices to prove the following inequalities:

$$P^{\text{eff-vmin}}_{\text{slow}}(t_0) \le P^{\text{eff-vmin}}_{\text{slow}}(t_0-1) + R^{\text{vup}}_{\text{slow}}(t_0-1), \quad (36)$$

$$\gamma^{\min}_{t_1}(D(1{:}t_0)) \le \gamma^{\min}_{t_1}(D(1{:}t_0-1)) + R^{\text{vup}}_{\text{slow}}(t_0-1), \text{ for all } t_1 \ge t_0. \quad (37)$$

Eqn. (36) can be proved as follows:

$$\text{RHS of (36)} = \max\left\{\max_{t_0-1<t'\le T}\{P^{\text{vmin}}_{\text{slow}}(t') - \sum_{s=t_0-1}^{t'-1} R^{\text{vup}}_{\text{slow}}(s)\}, \max_{t\le t'\le t_0-1}\{P^{\text{vmin}}_{\text{slow}}(t') - \sum_{s=t'}^{t_0-2} R^{\text{vdown}}_{\text{slow}}(s)\}\right\} + R^{\text{vup}}_{\text{slow}}(t_0-1)$$

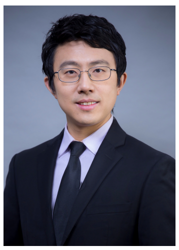
$$\ge \max\left\{\max_{t_0-1<t'\le T}\{P^{\text{vmin}}_{\text{slow}}(t') - \sum_{s=t_0}^{t'-1} R^{\text{vup}}_{\text{slow}}(s)\}, \max_{t\le t'\le t_0-1}\{P^{\text{vmin}}_{\text{slow}}(t') - \sum_{s=t'}^{t_0-1} R^{\text{vdown}}_{\text{slow}}(s)\}\right\} = P^{\text{eff-vmin}}_{\text{slow}}(t_0)$$

(by simple algebra and $R^{\text{vup}}_{\text{slow}}(t_0-1), R^{\text{vdown}}_{\text{slow}}(t_0-1) \ge 0$).

Eqn. (37) can be proved as follows:

$$
\begin{aligned}
&\gamma_{t_1}^{\min}(D(1{:}t_0)) \\
= &\max_{D(t_1)\in\mathcal{D}_{t_1|D(1:t_0)}} \{D(t_1)\} - \sum_{s=t_0}^{t_1-1} R_{\text{slow}}^{\text{vup}}(s) - r_{\text{fast}}^{\text{v+}}(t_1) \\
\le &\max_{D(t_1)\in\mathcal{D}_{t_1|D(1:t_0-1)}} \{D(t_1)\} - \sum_{s=t_0}^{t_1-1} R_{\text{slow}}^{\text{vup}}(s) - r_{\text{fast}}^{\text{v+}}(t_1) \\
&\qquad\qquad (\text{since } \mathcal{D}_{t_1|D(1:t_0)} \subset \mathcal{D}_{t_1|D(1:t_0-1)}) \\
= &\gamma_{t_1}^{\min}(D(1{:}t_0-1)) + R_{\text{slow}}^{\text{vup}}(t_0-1).
\end{aligned}
$$

The result of the theorem then follows.



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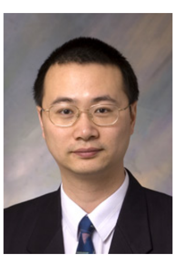

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